# 2011 GM VOLT EV PERFORMANCE SIMULATION

VXPhysics.com

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Mathcad 14 Simulation

## 2011 GM Volt Specifications

**Max Power:** $Power_{max} := 111kW \cdot 0.9$ $RPM_{max} := 12000$ Gear Ratio: $GR := 8$ $\eta_{axle} := 0.92$

Battery Energy: $Energy_{bat} := 16 \cdot kW \cdot hr$ $E_{useable} := 10.6kW \cdot hr$

$Power_{max} = 133.97 \cdot hp$

Max Motor Torque $T_{max} := 273 \cdot ft \cdot lbf$ Tire: 195/55R21 Radius: $r_{tire} := \frac{29.44}{2} in$ $F_{Motor\_Max} := \frac{T_{max} \cdot GR}{r_{tire}}$

Effective Mass Coefficient: $k_m := 1.08$ $car_{max\_g} := 1 \cdot g$ $k := 1000$

Curb/Gross Weight: $M_{curb} := 3140lbm$ $M_{gross} := M_{curb} + 170lbm = 3310 \cdot lbm$

Aerodynamic Drag Coeff (TM): $Cd := 0.28$ Average Wind Velocity: $Vw := 0 \cdot mph$ $g_{max} := \frac{T_{max} \cdot GR}{M_{gross} \cdot k_m \cdot r_{tire} \cdot g}$

Cross Wind Drag Coff: $Cd_{cw} := 0.000014$ Effective Cross Wind V: $V_{cw} := 0 \cdot mph$

Shape Correction Factor: $SCF := 0.85$ Vehicle Frontal Dimensions: $Af := 2.16 \cdot m^2$

Air Density, tire resistance: $\rho := 1.293 \cdot \frac{gm}{liter}$ Drag Frontal Area $Ad := Af \cdot SCF$ $Ad = 1.84 \cdot m^2$

Road Rolling Resistance: $RR_{road} := 0.002$ Tire Rolling Resist, Hys: $RR_{tire} := 0.011$ $T_{hys} := 0 \cdot \frac{sec}{m}$

## Macro Model of Motor Dynamics: Velocity of Tire is v

Angular Velocity Symbol, $\Omega$ (units of radians/second) $\Omega(\omega) := 2\pi 1000 \cdot \omega \cdot min^{-1}$ RPM/1000 Symbol, $\omega_k$ $RPM := min^{-1}$

Angular Vel $\Omega$ @Max Power: $\Omega_{Pmax} := Power_{max} \cdot T_{max}^{-1}$ $RPM_{Pmax} := \frac{\Omega_{Pmax}}{2 \cdot \pi}$ $RPM_{Pmax} = 2577.35 \cdot RPM$

Convert velocity to RPM $VtoRPM(v_v) := v_v \cdot (1000 \cdot 2 \cdot \pi \cdot r_{tire} \cdot RPM)^{-1}$ $\omega_{Pfall} := RPM_{Pmax} \cdot k^{-1} = 2.58 \cdot RPM$

Tire Velocity at Torque Fall: $v_{Tfall} := RPM_{Pmax} \cdot 2 \cdot \pi \cdot r_{tire} \cdot GR^{-1}$ $v_{Tfall} = 28.22 \cdot mph$

Tire Velocity to kRPM: $VtokR(v_t) := v_t \cdot (k \cdot 2 \cdot \pi \cdot r_{tire} \cdot RPM)^{-1}$ $\theta$ (radians): $\theta := atan(0.0)$

Road Resistance, Ft: $Ft(v_v) := M_{gross} \cdot g \cdot \left[T_{hys} \cdot v_v \cdot \sin(\theta) + (RR_{tire} + RR_{road}) \cdot \cos(\theta) + \sin(\theta)\right]$ $RPM_{pmax}$ for Max Power:

Air Drag Force, Fa: $Fa(v_v) := 0.5 \cdot \rho \cdot Ad \cdot \left[(v_v + Vw)^2 \cdot Cd + Cd_{cw} \cdot (V_{cw})^2\right]$

Total Opposing Force, Fo: $Fo(v_v) := Fa(v_v) + Ft(v_v)$ $Fo(60 \cdot mph) = 96.78 \cdot lbf$

Torque/Force Falloff Curve: $\omega_{kmax} := 15.8 \cdot RPM$ $T_{PLt}(\omega_k) := Power_{max} \cdot \Omega(\omega_k)^{-1}$ $T_{PLt}(55) = 12.79 \cdot ft \cdot lbf$

Tm is Torque of motor $T_m(\omega_k) := if(\omega_k \cdot RPM \geq \omega_{Pfall}, T_{PLt}(\omega_k), T_{max})$ $P_m(\omega_k) := T_m(\omega_k) \cdot k \cdot 2 \cdot \pi \cdot \omega_k \cdot RPM$

Fmot, Tractive Force from motor, not from slipping tires: $T_{mv}(v_t) := T_m(VtokR(v_t) \cdot GR)$ $F_{mot}(v_t) := \frac{GR}{r_{tire}} \cdot T_{mv}(v_t) \cdot \eta_{axle}$ $F_{PL}(v_t) := Power_{max} \cdot (v_t \cdot mph)^{-1}$

## Solve for Velocity, Acceleration, and Distance versus Time

**We are using Mathcad 14, a Computer Math Program, to do the Calculations: http://www.ptc.com/product/mathcad/free-trial**

### Newton's Third Law of Motion:

$$a_1(v) := \frac{F_{mot}(v) - Fo(v)}{k_m \cdot M_{gross}} \qquad a_{1Tmax} := \frac{T_{max} \cdot GR}{M_{gross} \cdot k_m \cdot r_{tire}} = 0.5 \cdot g$$

2011 Specs: 0 - 60 mph in 8.9

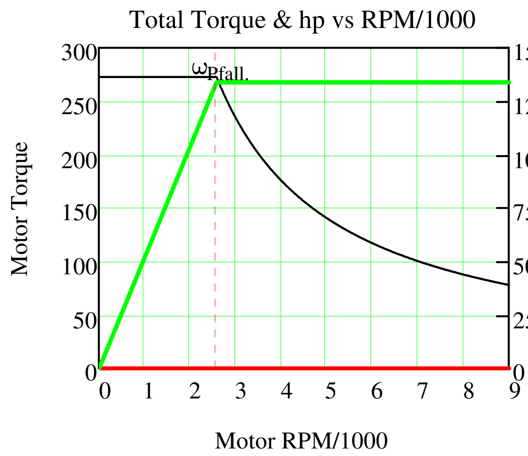
$$V := 0 \cdot mph \qquad vel_1(t) := root\left(t \cdot sec - \int_0^V \frac{mph}{a_1(V \cdot mph)} dV, V\right) \cdot mph \qquad time_{a1}(v) := \int_0^v \frac{1}{a_1(v)} dv$$

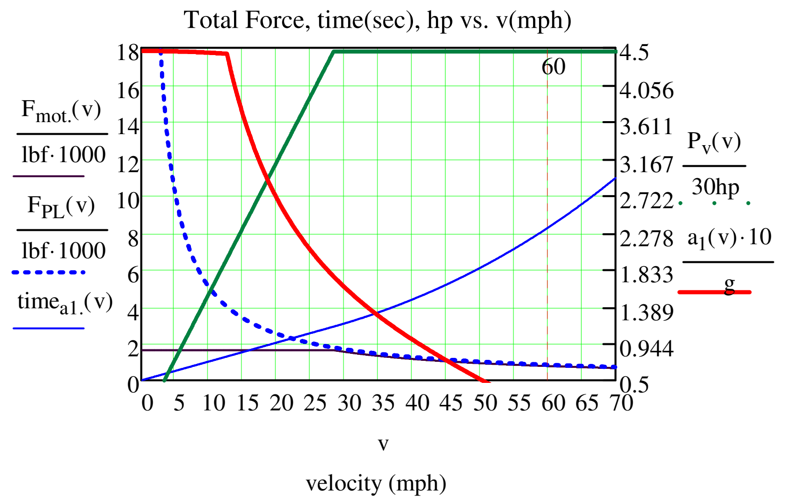
$time_{a1}(60mph) = 8.25\ s$

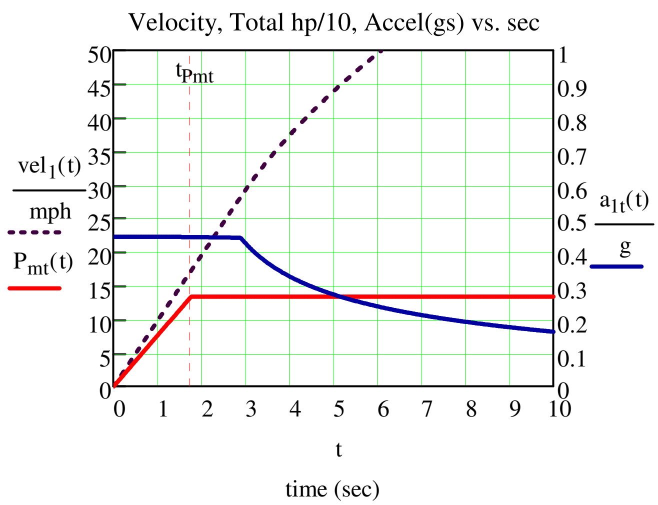
$vel_1(8.01) = 58.98 \cdot mph$

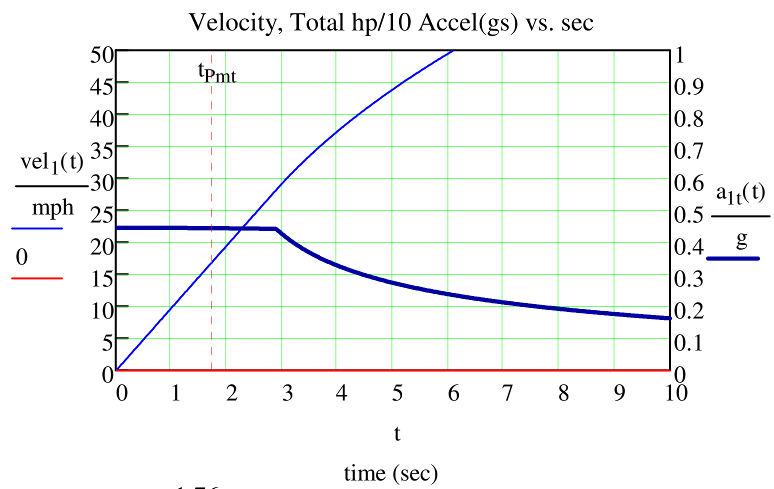
$a_{1t}(t) := a_1(vel_1(t))$

$$distance_1(t) := \int_0^t vel_1(t)\ \tau\ dt \qquad P_{mt}(t) := \frac{P_m\left(\frac{vel_1(t)}{r_{tire} \cdot GR} \cdot sec\right)}{10hp} \qquad P_v(v) := P_m\left(\frac{v \cdot mph \cdot min \cdot GR}{2 \cdot \pi \cdot r_{tire} \cdot k}\right) \qquad g_{spin} := 1.1$$




$t_{Pmt} = 1.76$

## ***Find the Single Charge. Highway Cruise Range for a Given Velocity and Final SOC***

**Driving Pattern/Profile:** Assume we cruise at constant speed, but start, stop, and regen break four times per hour

**Drive Train Power Efficiency - Battery Loss for Commanded Vehicle Velocity and Final State of Charge, $SOC_f$:**

**$SOC_f$ is 10% at recharge.** 400V HV battery idle power is Po. 12V battery gives Accessory Power. The Traction Inverter Efficiency - TInvE, HV Power Electronics at Idle Efficiency - IPEE, and Gear Power Efficiency - GPE are 92.5%, 95%, and 90%, respectively. Brake Regen efficiency of kinetic energy is 64%. Then the number of starts per hour as a function of velocity, NS, NumStarts(v, Po), is

$TInvE := 0.92$ $IPEE := 0.95$ $GPE := 0.95$ $Regen := 0.69$ Change in State of Charge = 1 - $SOC_f$

$$Power_{dissLoss}(v, P_o) := \frac{Fo(v)\cdot v}{TInvE\cdot GPE} + \frac{P_o\cdot watt}{IPEE} \qquad Energy_{accel}(v) := Power_{max}\cdot time(v\cdot mph)\cdot hr$$

NSo, NS are iterative converging estimates of total NumStarts per charge

$$NS_o(v) := 2\cdot\left[\frac{65mph}{(v + 0.1\cdot mph)}\right]^2 \qquad NS(v, P_o, SOC_f) := \frac{Energy_{bat}\cdot(1 - SOC_f) - NS_o(v)\cdot\left[\left[\frac{M_{gross}\cdot(v)^2}{2}(1 - Regen)\right]\right]}{Power_{dissLoss}(v, P_o)\cdot 15\cdot min}$$

$$NumStarts(v, P_o, SOC_f) := floor\left[\frac{Energy_{bat}\cdot(1 - SOC_f) - NS(v, P_o, SOC_f)\cdot\left[\frac{M_{gross}\cdot(v)^2}{2}(1 - Regen)\right]}{Power_{dissLoss}(v, P_o)\cdot 15\cdot min}\right]$$

$$Cruise\_Range(v, P_o, SOC_f) := \frac{\left[Energy_{bat}\cdot(1 - SOC_f) - NumStarts(v, P_o, SOC_f)\cdot\left[\frac{Regen\cdot M_{gross}\cdot(v)^2}{2}(1 - Regen)\right]\right]\cdot v}{Power_{dissLoss}(v, P_o)}$$

## Highway Cruise Range with Four Stops per Hour Estimate

$\text{Cruise\_Range}(30\cdot mph, 100, 0.1) = 108.25\cdot mi$

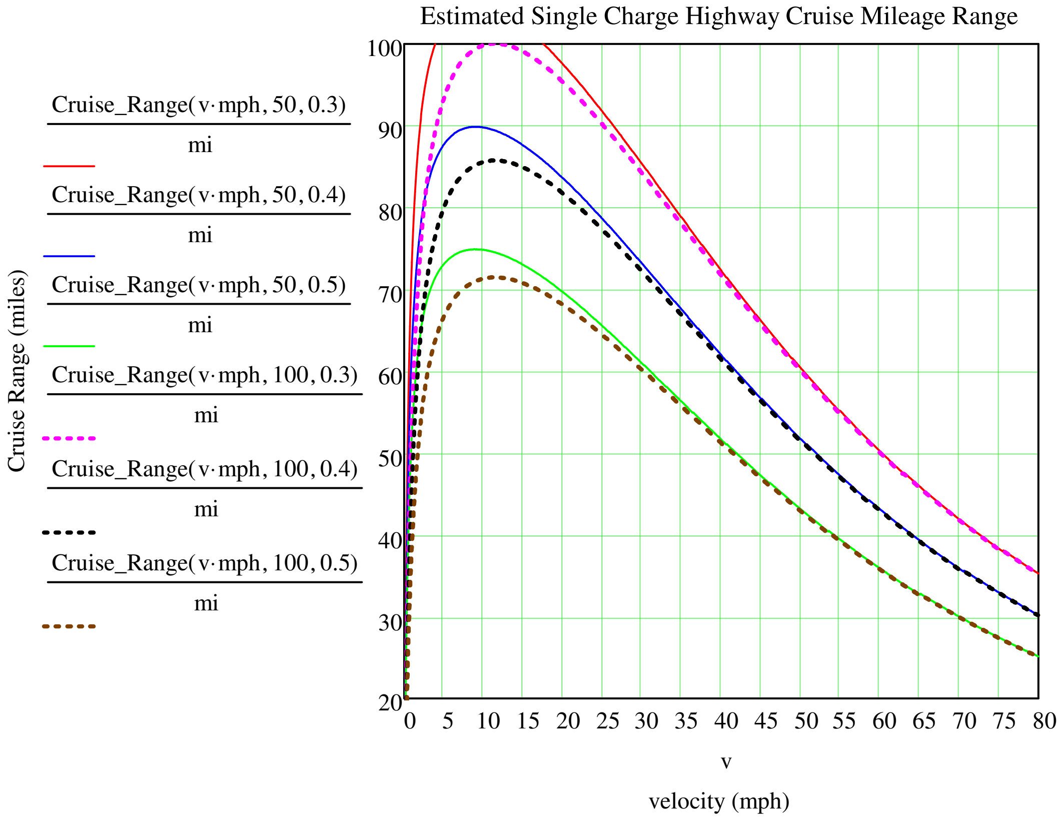
$\text{Cruise\_Range}(40\cdot mph, 100, 0.1) = 92.14\cdot mi$

$\text{Cruise\_Range}(50\cdot mph, 100, 0.1) = 77.14\cdot mi$

$\text{Cruise\_Range}(60\cdot mph, 100, 0.1) = 64.3\cdot mi$

$\text{Cruise\_Range}(70\cdot mph, 100, 0.1) = 53.67\cdot mi$

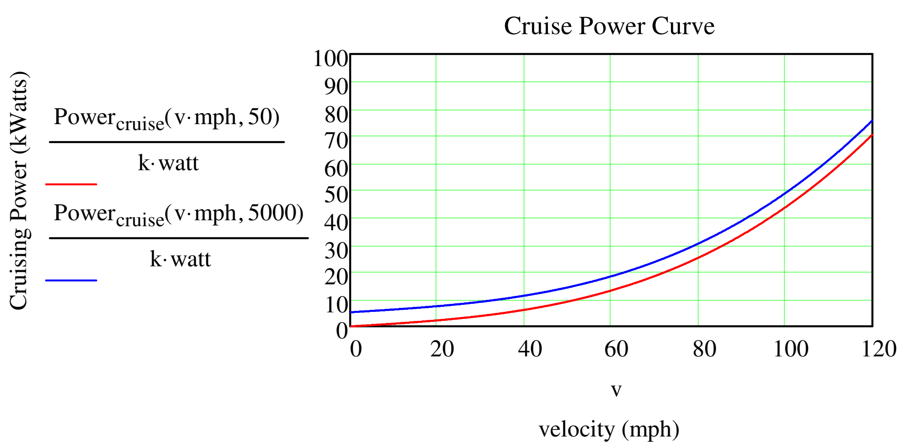
$\text{Cruise\_Range}(60\cdot mph, 200, 0) = 70.95\cdot mi$




## Opposing Force Power Dissipation

$Power_{cruise}(v, P_o) := Power_{dissLoss}(v, P_o)$

$Power_{cruise}(60\cdot mph, 500) = 1.37 \times 10^{4}\cdot watt$



# Find Mileage Range: Use 3 Different EPA Driving Schedules

**Regen Efficiency Curve vs Decel (g)**: $Regen := 0.75$ MinStateOfCharge: $SOC_{min} := 0.5$

**Algorithm to Calculate Range, Range(P,fHz), 100% Battery Discharge, Driving Profile Velocity/Time File, P and Sampling Rate, fHz**

$$
\begin{aligned}
Range(P, f_{Hz}) := \; & Ebat \leftarrow E_{diss} \leftarrow v_{old} \leftarrow 0 \\
& n \leftarrow -1 \\
& N \leftarrow rows(P) - 1 \\
& \text{while } \left[ E_{diss} < (1 - SOC_{min}) \cdot \frac{Energy_{bat}}{kW \cdot hr} \right] \\
& \quad n \leftarrow n + 1 \\
& \quad t \leftarrow mod(n, N) \\
& \quad v \leftarrow P_t \\
& \quad P_{accel} \leftarrow \frac{k_m \cdot M_{gross} \cdot (v^2 - v_{old}{}^2) \cdot \frac{mph \cdot f_{Hz}}{sec} mph}{TInvE \cdot GPE \cdot 2} \quad \text{if } v > v_{old} \\
& \quad P_{accel} \leftarrow k_m \cdot M_{gross} \cdot (v^2 - v_{old}{}^2) \cdot \frac{mph \cdot f_{Hz}}{2sec} mph \cdot Regen \quad \text{otherwise} \\
& \quad E_{diss} \leftarrow E_{diss} + \frac{(Power_{dissLoss}(v \cdot mph, 100) + P_{accel}) \cdot sec}{kW \cdot hr \cdot f_{Hz}} \\
& \quad v_{old} \leftarrow v \\
& \quad Ebat_n \leftarrow E_{diss} \\
& Range \leftarrow \sum_{m=0}^{n} \frac{(P_{mod(m,N)} + P_{mod(m+1,N)}) \cdot mph \cdot sec}{2 \cdot mi \cdot f_{Hz}}
\end{aligned}
$$

If decelerating, charge battery with Regen fraction of energy.

**Read US06 and FTP Dynamometer Drive Profile Files**

**Refer to**: http://www.epa.gov/nvfel/testing/dynamometer.htm

The US06 cycle represents an 8.01 mile (12.8 km) route with an average speed of 48.4 miles/h (77.9 km/h), maximum speed 80.3 miles/h (129.2 km/h), and a duration of 596 seconds. Sampling can be either 1 Hz or 10Hz

The **Federal Test Procedure (FTP)** is composed of the UDDS followed by the first 505 seconds of the UDDS. It is often called the EPA75. 10 Hz Sampling data is named FP10 and HY10 for the Highway schedule.

FTPF := READPRN("FedTestProc.txt") $\quad t := FTPF^{\langle 0 \rangle} \quad FTP := FTPF^{\langle 1 \rangle} \quad rows(FTP) = 1875$

UDDSF := READPRN("uddscol.txt") $\quad UDDS := UDDSF^{\langle 1 \rangle} \quad rows(UDDS) = 1370$

HWYF := READPRN("hwycol.txt") $\quad HWY := HWYF^{\langle 1 \rangle} \quad R_{hwy} := rows(HWY)$

FP10 := READPRN("FTP10Hz.TXT") $\quad FTP10V := submatrix(FP10, 0, rows(FP10) - 1, 1, cols(FP10) - 1)$

HY10 := READPRN("HWY10Hz.TXT") $\quad HWY10V := submatrix(HY10, 0, rows(HY10) - 1, 1, cols(HY10) - 1)$

US06F := READPRN("US06PROFILE.TXT") $\quad Time := US06F^{\langle 0 \rangle} \quad US06 := US06F^{\langle 1 \rangle} \quad n_6 := 0..598$

$r1 := 0..rows(HY10) \cdot 10 - 1 \quad HWY10_{r1} := HWY10V_{ceil\left(\frac{r1+1}{10}\right)-1,\, mod(r1, 10)}$

## Using EPA Profiles and above Range Program, Calculate Tesla EV Range for EPA Profiles

$Range_{US06} := Range(US06, 1)$ $Range_{FTP} := Range(FTP, 1)$ $Range_{HWY} := Range(HWY, 1)$

**EPA 2008 Cycle MPG Fuel Economy Least Squares Fit Regression for Range**

$$MPG_{city} := \frac{1}{\left(0.003259 + \frac{1.18053}{Range_{FTP}}\right)} \qquad MPG_{hwy} := \frac{1}{0.001376 + \frac{1.3466}{Range_{HWY}}}$$

$$MPG_{epa} := 0.55 \cdot MPG_{city} + 0.45 \cdot MPG_{hwy}$$

## Single Charge EPA Range Calculations: Federal Test Procedure (FTP), Highway, and US06

**Model Validation:** **Published EPA Range is 40 miles**

EPA uses a combined City and Highway weighted average of 55% city and 45% Highway, $Range_{combined}$

$Range_{FTP} = 40.9$ $Range_{HWY} = 37.94$ $Range_{US06} = 27.03$

$$Range_{combined} := 0.55 \cdot Range_{FTP} + 0.45 \cdot Range_{HWY} = 39.57$$

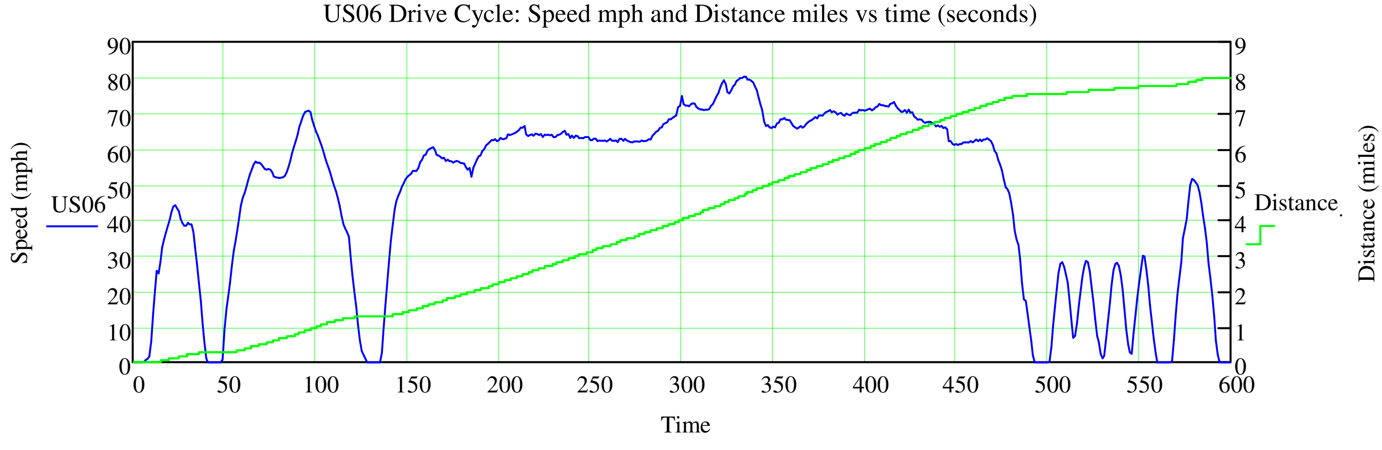
$MPG_{city} = 31.13$ $MPG_{hwy} = 27.13$ $MPG_{epa} = 29.33$

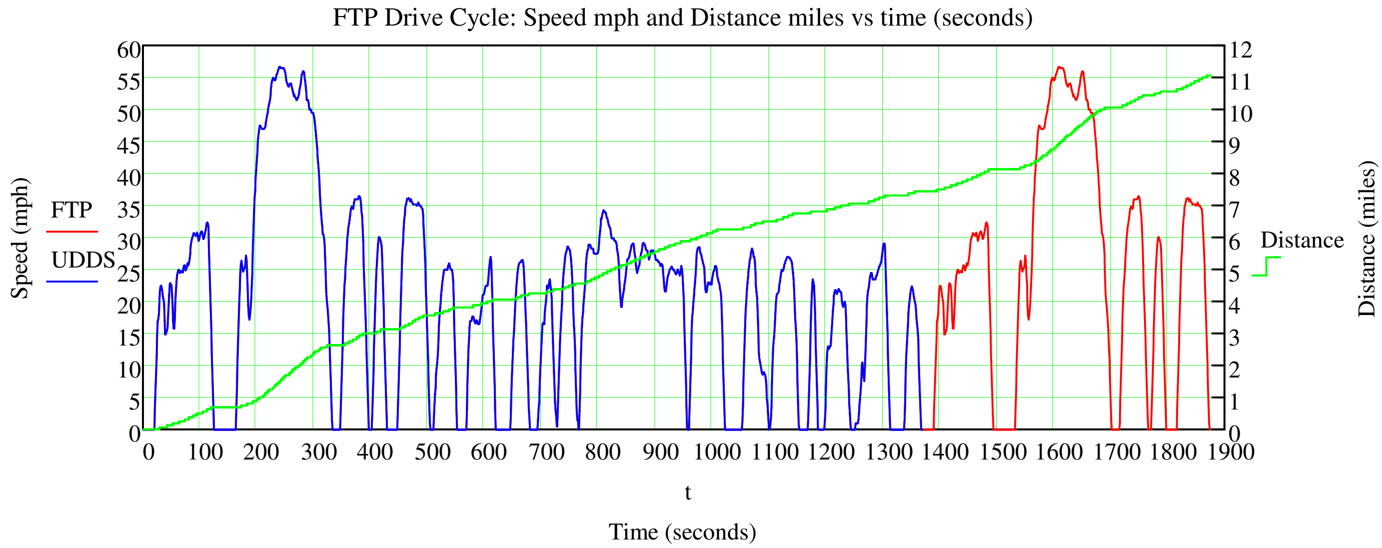
$r := 0 .. rows(FTP) - 1$ $Distance_r := \sum_{r=0}^{r} FTP_r \cdot \frac{1}{60 \cdot 60}$ $rr := 0 .. rows(US06) - 1$ $Distance._{rr} := \sum_{rr=0}^{rr} US06_{rr} \cdot \frac{1}{60 \cdot 60}$

$max(Distance) = 11.04$

$max(Distance) = 11.04$ $max(Distance.) = 8.01$

**Plots of EPA Dynamometer Vehicle Testing Profiles**






$$r := 0 .. \text{rows}(HWY) - 1 \qquad \text{Distance}_r := \sum_{r=0}^{r} HWY_r \cdot \frac{1}{60 \cdot 60}$$