NISSAN LEAF EV PERFORMANCE SIMULATION

VXPhysics.com

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Mathcad 14 Simulation

Nissan Leaf Specifications

Max Power: $Power_{max} := 107hp \cdot 0.9$ $RPM_{max} := 12000$ Gear Ratio: $GR := 7.938$ $\eta_{axle} := 0.95$

$Power_{max} = 96.3\,hp$

Max Motor Torque $T_{max} := 207 \cdot ft \cdot lbf$ Tire: 205/55R16 Radius: $r_{tire} := \frac{24.88}{2} in$ $F_{Motor\_Max} := \frac{T_{max} \cdot GR}{r_{tire}}$

Effective Mass Coefficient: $k_m := 1.08$ $car_{max\_g} := 1 \cdot g$ $k := 1000$

Curb/Gross Weight: $M_{curb} := 3256lbm$ $M_{gross} := M_{curb} + 170lbm = 3426 \cdot lbm$

Aerodynamic Drag Coeff (TM): $Cd := 0.29$ Average Wind Velocity: $Vw := 0 \cdot mph$ $g_{max} := \frac{T_{max} \cdot GR}{M_{gross} \cdot k_m \cdot r_{tire} \cdot g}$

Cross Wind Drag Coff: $Cd_{cw} := 0.000014$ Effective Cross Wind V: $V_{cw} := 0 \cdot mph$

Shape Correction Factor: $SCF := 1$ Vehicle Frontal Dimensions: $Af := 2.27 \cdot m^2$ $M_{batt} := 600lb$

Air Density, tire resistance: $\rho := 1.293 \cdot \frac{gm}{liter}$ Drag Frontal Area $Ad := Af \cdot SCF$ $Ad = 2.27 \cdot m^2$

Road Rolling Resistance: $RR_{road} := 0.002$ Tire Rolling Resist, Hys: $RR_{tire} := 0.011$ $T_{hys} := 0 \cdot \frac{sec}{m}$

Macro Model of Motor Dynamics: Velocity of Tire is v

Angular Velocity Symbol, Ω (units of radians/second) $\Omega(\omega) := 2\pi 1000 \cdot \omega \cdot min^{-1}$ RPM/1000 Symbol, ω_k $RPM := min^{-1}$

Angular Vel Ω @Max Power: $\Omega_{Pmax} := Power_{max} \cdot T_{max}^{-1}$ $RPM_{Pmax} := \frac{\Omega_{Pmax}}{2 \cdot \pi}$ $RPM_{Pmax} = 2443.37 \cdot RPM$

Convert velocity to RPM $VtoRPM(v_v) := v_v \cdot (1000 \cdot 2 \cdot \pi \cdot r_{tire} \cdot RPM)^{-1}$ $\omega_{Pfall} := RPM_{Pmax} \cdot k^{-1} = 2.44 \cdot RPM$

Tire Velocity at Torque Fall: $v_{Tfall} := RPM_{Pmax} \cdot 2 \cdot \pi \cdot r_{tire} \cdot GR^{-1}$ $v_{Tfall} = 22.78 \cdot mph$

Tire Velocity to kRPM: $VtokR(v_t) := v_t \cdot (k \cdot 2 \cdot \pi \cdot r_{tire} \cdot RPM)^{-1}$ θ (radians): $\theta := atan(0.0)$

Road Resistance, Ft: $Ft(v_v) := M_{gross} \cdot g \cdot [T_{hys} \cdot v_v \cdot sin(\theta) + (RR_{tire} + RR_{road}) \cdot cos(\theta) + sin(\theta)]$ RPM_{pmax} for Max Power:

Air Drag Force, Fa: $Fa(v_v) := 0.5 \cdot \rho \cdot Ad \cdot \left[(v_v + Vw)^2 \cdot Cd + Cd_{cw} \cdot (V_{cw})^2\right]$

Total Opposing Force, Fo: $Fo(v_v) := Fa(v_v) + Ft(v_v)$ $Fo(60 \cdot mph) = 113.37 \cdot lbf$

Torque/Force Falloff Curve: $\omega_{kmax} := 15.8 \cdot RPM$ $T_{PLt}(\omega_k) := Power_{max} \cdot \Omega(\omega_k)^{-1}$ $T_{PLt}(55) = 9.2 \cdot ft \cdot lbf$

Tm is Torque of motor $T_m(\omega_k) := if(\omega_k \cdot RPM \geq \omega_{Pfall}, T_{PLt}(\omega_k), T_{max})$ $P_m(\omega_k) := T_m(\omega_k) \cdot k \cdot 2 \cdot \pi \cdot \omega_k \cdot RPM$

Fmot, Tractive Force from motor, not from slipping tires: $T_{mv}(v_t) := T_m(VtokR(v_t) \cdot GR)$ $F_{mot}(v_t) := \frac{GR}{r_{tire}} \cdot T_{mv}(v_t) \cdot \eta_{axle}$ $F_{PL}(v_t) := Power_{max} \cdot (v_t \cdot mph)^{-1}$

Solve for Velocity, Acceleration, and Distance versus Time

We are using Mathcad 14, a Computer Math Program, to do the Calculations: http://www.ptc.com/product/mathcad/free-trial

Newton's Third Law of Motion:

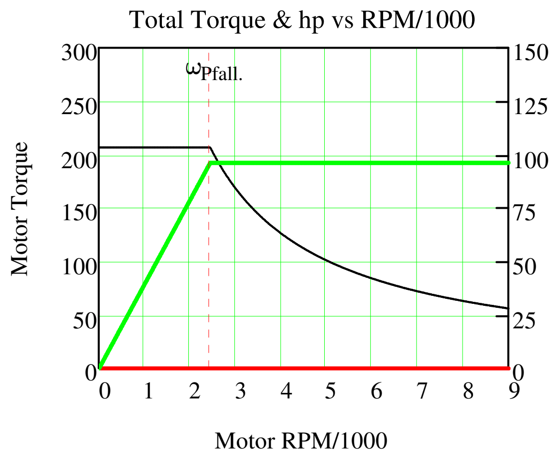
$a_1(v) := \frac{F_{mot}(v) - Fo(v)}{k_m \cdot M_{gross}}$ $a_1(0) = 0.39 \cdot g$ $a_{1Tmax} := \frac{T_{max} \cdot GR}{M_{gross} \cdot k_m \cdot r_{tire}} = 0.43 \cdot g$

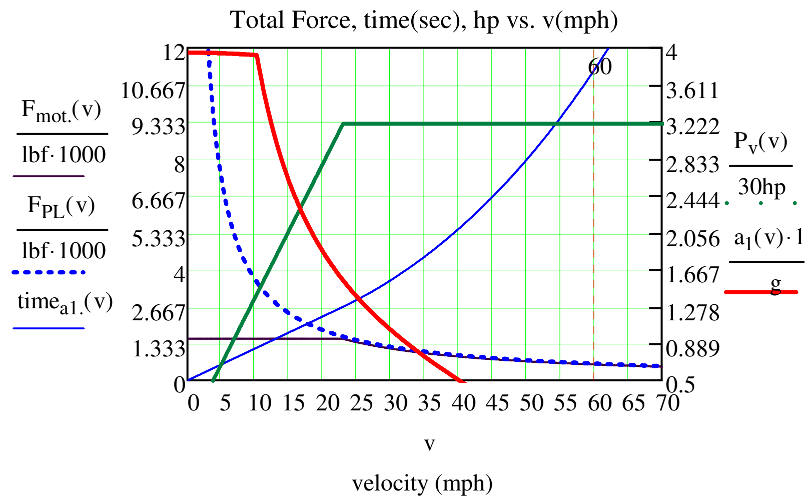
2016 Specs: 0 - 60 mph in 10.2 s

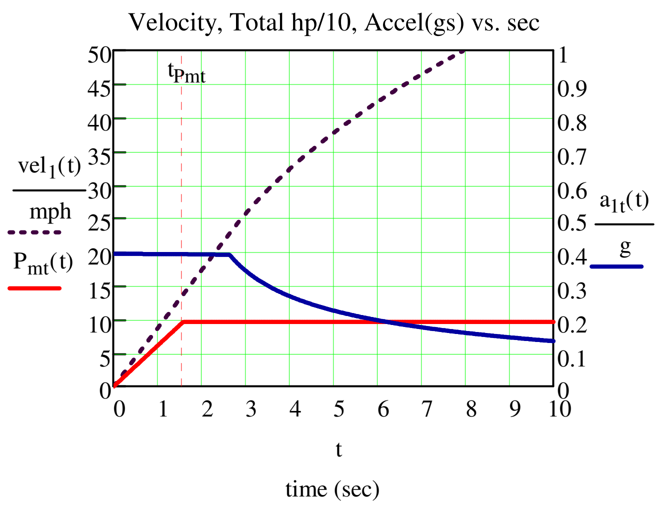
$V := 0 \cdot mph$ $vel_1(t) := root\left(t \cdot sec - \int_0^V \frac{mph}{a_1(V \cdot mph)} dV, V\right) \cdot mph$ $time_{a1}(v) := \int_0^v \frac{1}{a_1(v)} dv$ $time_{a1}(60mph) = 11.2\,s$

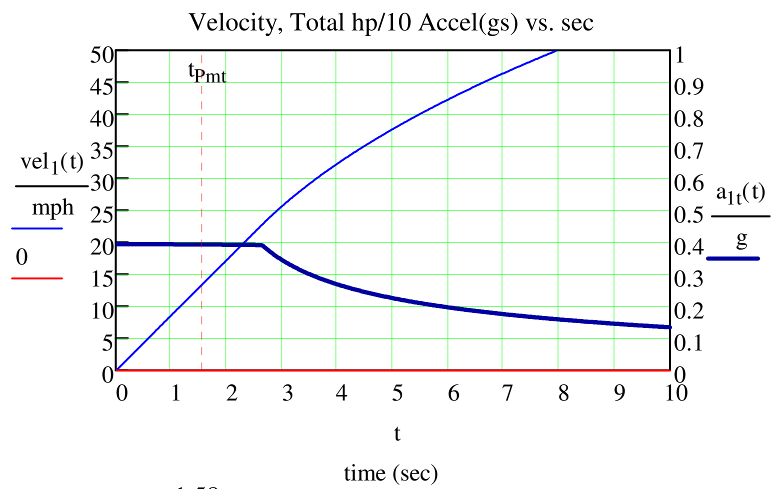
$a_{1t}(t) := a_1(vel_1(t))$

$distance_1(t) := \int_0^t vel_1(t)\,\tau\,dt$ $P_{mt}(t) := \frac{P_m\left(\frac{vel_1(t)}{r_{tire} \cdot GR} \cdot sec\right)}{10hp}$ $P_v(v) := P_m\left(\frac{v \cdot mph \cdot min \cdot GR}{2 \cdot \pi \cdot r_{tire} \cdot k}\right)$ $g_{spin} := 1.1$









$t_{Pmt} = 1.58$

Find the Single Charge. Highway Cruise Range for a Given Velocity and Final SOC

Driving Pattern/Profile: Assume we cruise at constant speed, but start, stop, and regen break four times per hour

Drive Train Power Efficiency - Battery Loss for Commanded Vehicle Velocity and Final State of Charge, SOC_f:

SOC_f is 10% at recharge. 400V HV battery idle power is Po. 12V battery gives Accessory Power. The Traction Inverter Efficiency - TInvE, HV Power Electronics at Idle Efficiency - IPEE, and Gear Power Efficiency - GPE are 92.5%, 95%, and 90%, respectively. Brake Regen efficiency of kinetic energy is 64%. Then the number of starts per hour as a function of velocity, NS, NumStarts(v, Po), is

$TInvE := 0.92$ $IPEE := 0.95$ $GPE := 0.95$ $Regen := 0.69$ Change in State of Charge = 1 - SOC_f

$Energy_{bat} := 24kW \cdot hr$

$$Power_{dissLoss}(v, P_o) := \frac{Fo(v) \cdot v}{TInvE \cdot GPE} + \frac{P_o \cdot watt}{IPEE}$$

$$Energy_{accel}(v) := Power_{max} \cdot time(v \cdot mph) \cdot hr$$

NSo, NS are iterative converging estimates of total NumStarts per charge

$$NS_o(v) := 2 \cdot \left[\frac{65mph}{(v + 0.1 \cdot mph)}\right]^2$$

$$NS(v, P_o, SOC_f) := \frac{Energy_{bat} \cdot (1 - SOC_f) - NS_o(v) \cdot \left[\left[\frac{M_{gross} \cdot (v)^2}{2} (1 - Regen)\right]\right]}{Power_{dissLoss}(v, P_o) \cdot 15 \cdot min}$$

$$NumStarts(v, P_o, SOC_f) := floor\left[\frac{Energy_{bat} \cdot (1 - SOC_f) - NS(v, P_o, SOC_f) \cdot \left[\frac{M_{gross} \cdot (v)^2}{2} (1 - Regen)\right]}{Power_{dissLoss}(v, P_o) \cdot 15 \cdot min}\right]$$

$$Cruise\_Range(v, P_o, SOC_f) := \frac{\left[Energy_{bat} \cdot (1 - SOC_f) - NumStarts(v, P_o, SOC_f) \cdot \left[\frac{Regen \cdot M_{gross} \cdot (v)^2}{2} (1 - Regen)\right]\right] \cdot v}{Power_{dissLoss}(v, P_o)}$$

Highway Cruise Range with Four Stops per Hour Estimate

$\text{Cruise\_Range}(30\cdot\text{mph}, 100, 0.1) = 148.91\cdot\text{mi}$

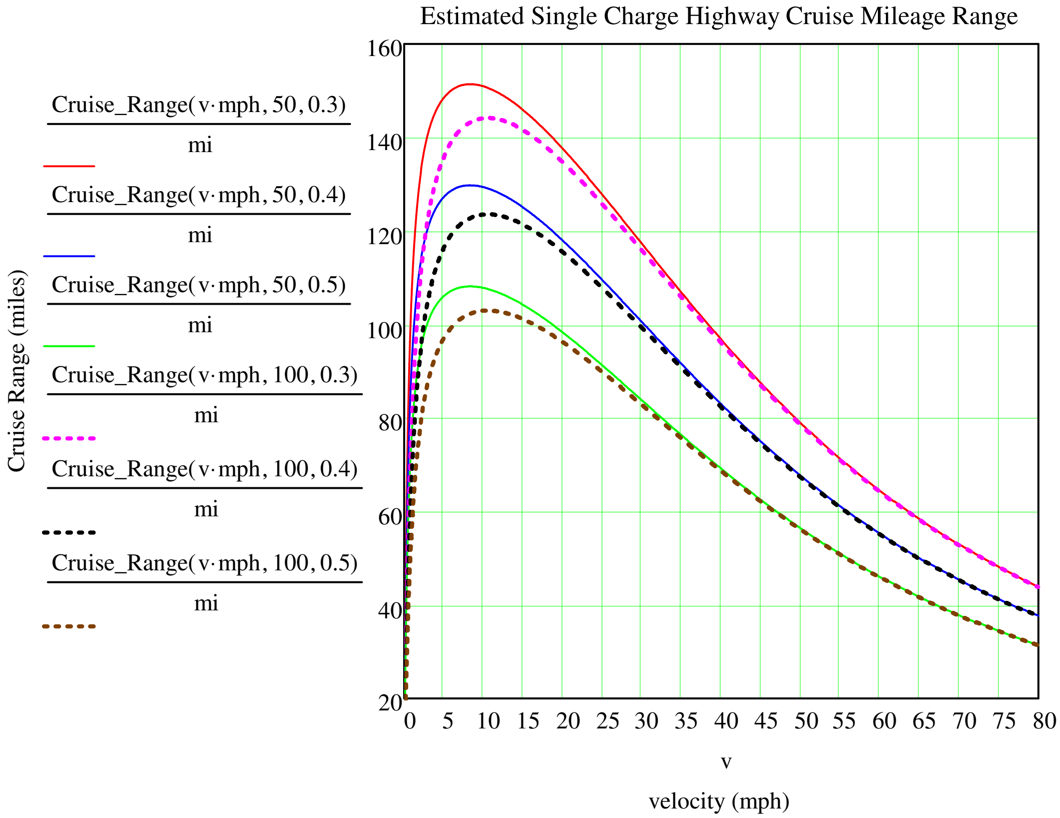
$\text{Cruise\_Range}(40\cdot\text{mph}, 100, 0.1) = 123.43\cdot\text{mi}$

$\text{Cruise\_Range}(50\cdot\text{mph}, 100, 0.1) = 100.92\cdot\text{mi}$

$\text{Cruise\_Range}(60\cdot\text{mph}, 100, 0.1) = 82.53\cdot\text{mi}$

$\text{Cruise\_Range}(70\cdot\text{mph}, 100, 0.1) = 67.92\cdot\text{mi}$

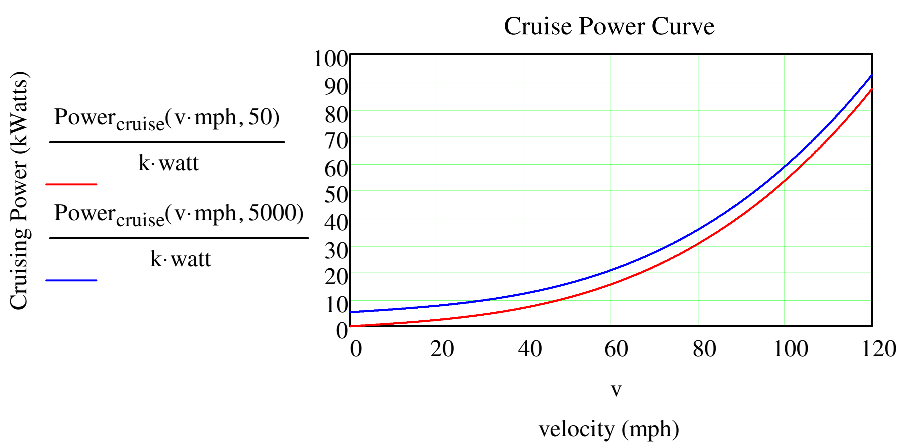
$\text{Cruise\_Range}(60\cdot\text{mph}, 200, 0) = 91.03\cdot\text{mi}$



Opposing Force Power Dissipation

$\text{Power}_{\text{cruise}}(v, P_o) := \text{Power}_{\text{dissLoss}}(v, P_o)$

$\text{Power}_{\text{cruise}}(60\cdot\text{mph}, 500) = 1.6\times 10^{4}\cdot\text{watt}$



Find Mileage Range: Use 3 Different EPA Driving Schedules

Regen Efficiency Curve vs Decel (g): $Regen := 0.75$

State of Charge Range: 95% to 5% (Turtle Mode)

$SOC_{range} := 0.92$

Algorithm to Calculate Range, Range(P,fHz), 100% Battery Discharge, Driving Profile Velocity/Time File, P and Sampling Rate, fHz

$$
\begin{aligned}
Range(E_{bat}, P, f_{Hz}) := \; & Ebat \leftarrow E_{diss} \leftarrow v_{old} \leftarrow 0 \\
& n \leftarrow -1 \\
& N \leftarrow rows(P) - 1 \\
& \text{while } \left(E_{diss} < SOC_{range} \cdot E_{bat}\right) \\
& \quad n \leftarrow n + 1 \\
& \quad t \leftarrow mod(n, N) \\
& \quad v \leftarrow P_t \\
& \quad P_{accel} \leftarrow \frac{k_m \cdot M_{gross} \cdot \left(v^2 - v_{old}^2\right) \cdot \frac{mph \cdot f_{Hz}}{sec} mph}{TInvE \cdot GPE \cdot 2} \quad \text{if } v > v_{old} \\
& \quad P_{accel} \leftarrow k_m \cdot M_{gross} \cdot \left(v^2 - v_{old}^2\right) \cdot \frac{mph \cdot f_{Hz}}{2sec} mph \cdot Regen \quad \text{otherwise} \\
& \quad E_{diss} \leftarrow E_{diss} + \frac{\left(Power_{dissLoss}(v \cdot mph, 100) + P_{accel}\right) \cdot sec}{kW \cdot hr \cdot f_{Hz}} \\
& \quad v_{old} \leftarrow v \\
& \quad Ebat_n \leftarrow E_{diss} \\
& Range \leftarrow \sum_{m=0}^{n} \frac{\left(P_{mod(m,N)} + P_{mod(m+1,N)}\right) \cdot mph \cdot sec}{2 \cdot mi \cdot f_{Hz}}
\end{aligned}
$$

If decelerating, charge battery with Regen fraction of energy.

Read US06 and FTP Dynamometer Drive Profile Files

Refer to: http://www.epa.gov/nvfel/testing/dynamometer.htm

The US06 cycle represents an 8.01 mile (12.8 km) route with an average speed of 48.4 miles/h (77.9 km/h), maximum speed 80.3 miles/h (129.2 km/h), and a duration of 596 seconds. Sampling can be either 1 Hz or 10Hz

The **Federal Test Procedure (FTP)** is composed of the UDDS followed by the first 505 seconds of the UDDS. It is often called the EPA75. 10 Hz Sampling data is named FP10 and HY10 for the Highway schedule.

FTPF := READPRN("FedTestProc.txt") $\quad t := FTPF^{\langle 0 \rangle} \quad FTP := FTPF^{\langle 1 \rangle} \quad rows(FTP) = 1875$

UDDSF := READPRN("uddscol.txt") $\quad UDDS := UDDSF^{\langle 1 \rangle} \quad rows(UDDS) = 1370$

HWYF := READPRN("hwycol.txt") $\quad HWY := HWYF^{\langle 1 \rangle} \quad R_{hwy} := rows(HWY)$

FP10 := READPRN("FTP10Hz.TXT") $\quad FTP10V := submatrix(FP10, 0, rows(FP10) - 1, 1, cols(FP10) - 1)$

HY10 := READPRN("HWY10Hz.TXT") $\quad HWY10V := submatrix(HY10, 0, rows(HY10) - 1, 1, cols(HY10) - 1)$

US06F := READPRN("US06PROFILE.TXT") $\quad Time := US06F^{\langle 0 \rangle} \quad US06 := US06F^{\langle 1 \rangle} \quad n_6 := 0..598$

$r1 := 0..rows(HY10) \cdot 10 - 1 \quad HWY10_{r1} := HWY10V_{ceil\left(\frac{r1+1}{10}\right)-1,\, mod(r1, 10)}$

Using EPA Profiles and above Range Program, Calculate Tesla EV Range for EPA Profiles

$Range24_{US06} := Range(24, US06, 1)$ $Range24_{FTP} := Range(24, FTP, 1)$ $Range24_{HWY} := Range(24, HWY, 1)$

$Range30_{US06} := Range(30, US06, 1)$ $Range30_{FTP} := Range(30, FTP, 1)$ $Range30_{HWY} := Range(30, HWY, 1)$

EPA 2008 Cycle MPG Fuel Economy Least Squares Fit Regression for Range

$$MPG_{city} := \frac{1}{\left(0.003259 + \frac{1.18053}{Range24_{FTP}}\right)} \qquad MPG_{hwy} := \frac{1}{0.001376 + \frac{1.3466}{Range24_{HWY}}}$$

$$MPG_{epa} := 0.55 \cdot MPG_{city} + 0.45 \cdot MPG_{hwy}$$

Single Charge EPA Range Calculations: Federal Test Procedure (FTP), Highway, and US06

Model Validation:

Published EPA Range for 24 kW hr Battery is "Up to" 84 miles
Published EPA Range for 30 kW hr Battery is "Up to" 107 miles

EPA uses a combined City and Highway weighted average of 55% city and 45% Highway, $Range_{combined}$

Range Simulation for 24 kW hr Battery

$Range24_{FTP} = 103.07$ $Range24_{HWY} = 92.89$ $Range24_{US06} = 65.88$

$Range_{combined} := 0.55 \cdot Range24_{FTP} + 0.45 \cdot Range24_{HWY} = 98.49$

Range Simulation for 30 kW hr Battery

$Range30_{FTP} = 129.44$ $Range30_{HWY} = 116.46$ $Range30_{US06} = 82.39$

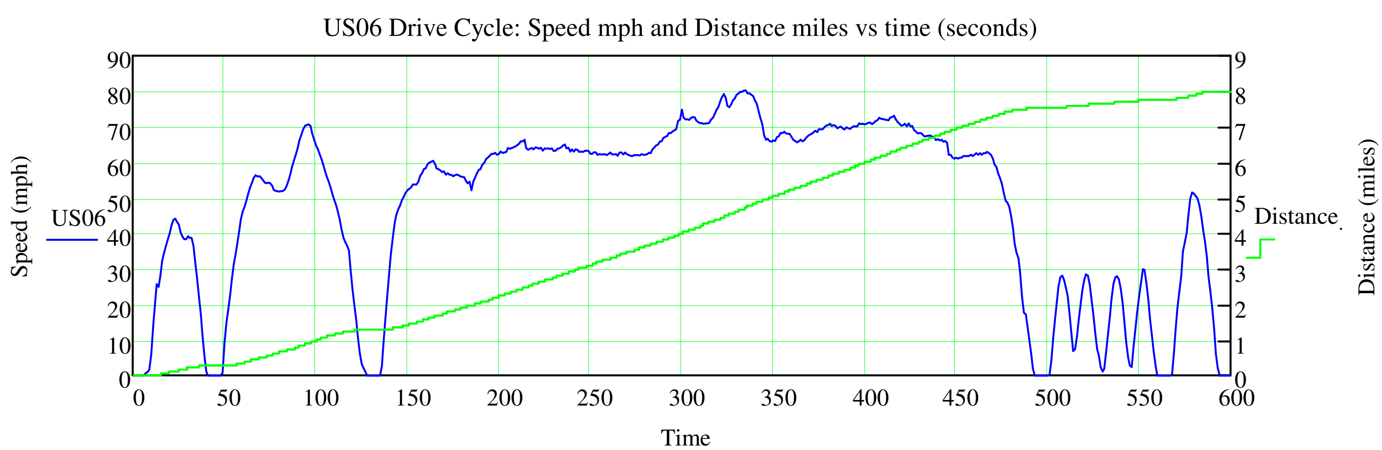
$Range_{combined} := 0.55 \cdot Range30_{FTP} + 0.45 \cdot Range30_{HWY} = 123.6$

$MPG_{city} = 67.97$ $MPG_{hwy} = 63$ $MPG_{epa} = 65.73$

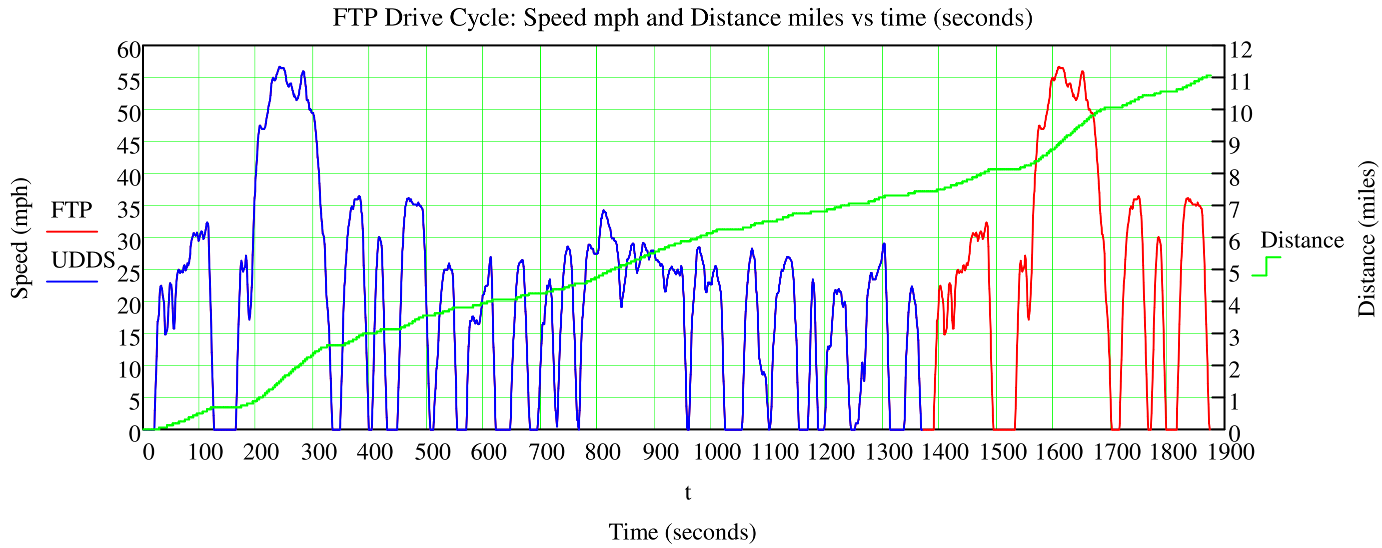
$r := 0 .. rows(FTP) - 1$ $\quad Distance_r := \sum_{r=0}^{r} FTP_r \cdot \frac{1}{60 \cdot 60}$ $\quad rr := 0 .. rows(US06) - 1$ $\quad Distance._{rr} := \sum_{rr=0}^{rr} US06_{rr} \cdot \frac{1}{60 \cdot 60}$

$max(Distance) = 11.04$ $\quad max(Distance) = 11.04$ $\quad max(Distance.) = 8.01$

Plots of EPA Dynamometer Vehicle Testing Profiles



Time (seconds)



$$r := 0 .. \text{rows}(HWY) - 1 \qquad Distance_r := \sum_{r=0}^{r} HWY_r \cdot \frac{1}{60 \cdot 60}$$