Chatter Detection in Machine Milling by Signal Processing:
Empirical Mode Decomposition, Hilbert Transform, & FFT Spectrum

Chatter is generally defined as self-generated vibrations from the interaction between tool and workpiece. It is the most critical vibration in machining operations and can decrease the surface quality and cause the premature tool wear. Chatter creates erroneous vibrations on the workpiece surface.

This mathcad worksheet is a signal processing technique of chatter detection for boring bar based on the Hilbert–Huang Transform (HHT). HHT is suitable for the analysis of non-stationary and non-linear signals. The flow of HHT for processing chatter signal and the principle are introduced. The signals are decomposed into several intrinsic mode functions (IMFs) using empirical mode decomposition (EMD). The Hilbert transform is then applied on each IMF to obtain the instantaneous frequencies with time and their amplitudes. Finally, the marginal and the Hilbert spectrums of strain signals were produced using selected IMFs.

Methodology: ***Chatter Identification of Face Milling Operation via Time-Frequency and Fourier Analysis,***
Ching-Chih Wei, Meng-Kun Liu, and Guo-Hua Huang AuSMT Vol. 6 No. 1 (2016)

Note: The Array Origin in the referenced worksheets below is set to 1.

Reference:C:\Users\Tom\Documents__Mathcad\Signal Analysis\EMD Extrema.xmcd(R)

Reference:C:\Users\Tom\Documents__Mathcad\Signal Analysis\EMD HHT Function.xmcd(R)

Read Two Sets of Data (A and B having Different depths of Cut) in 4 Columns (Time-microsec, Vibration-mv)

$\text{Milling\_Strain} := \text{READPRN}(\text{"Machine Milling Vibration Data.txt"})$ $\quad \text{time} := \text{Milling\_Strain}^{\langle 1 \rangle}$ $\quad S := \text{Milling\_Strain}^{\langle 2 \rangle}$

$\max(S) = 18$ $\quad \min(S) = -21$ $\quad RT := \text{rows}(S) = 1303$

$\text{rows}(\text{extrema}(S)) = 1003$ $\quad HHT := \text{eemd}(S, 0, 1)$ $\quad \text{cols}(HHT) = 11$ $\quad n := 2..11$

$r := 1..RT$ $\quad X_r := r$ $\quad \text{Residual} := HHT^{\langle 6 \rangle} + HHT^{\langle 7 \rangle} + HHT^{\langle 8 \rangle} + HHT^{\langle 9 \rangle} + HHT^{\langle 10 \rangle} + HHT^{\langle 11 \rangle}$

$\text{Ext} := \text{extrema}(S)$ $\quad \text{RExt} := \text{rows}(\text{Ext})$ $\quad \text{spmax} := \text{submatrix}\left(\text{Ext}, 1, \text{Ext}_{\text{RExt},2}, 1, 2\right)$ $\quad \text{Ext}_{\text{RExt}-1,2} = -4.5$

$\text{spmin} := \text{submatrix}\left(\text{Ext}, \text{Ext}_{\text{RExt},2} + 1, \text{RExt} - 2, 1, 2\right)$ $\quad up_{sp} := \text{cspline}\left(\text{spmax}^{\langle 1 \rangle}, \text{spmax}^{\langle 2 \rangle}\right)$

$\text{upper} := \text{interp}\left(up_{sp}, \text{spmax}^{\langle 1 \rangle}, \text{spmax}^{\langle 2 \rangle}, X\right)$

$low_{sp} := \text{cspline}\left(\text{spmin}^{\langle 1 \rangle}, \text{spmin}^{\langle 2 \rangle}\right)$ $\quad \text{lower} := \text{interp}\left(low_{sp}, \text{spmin}^{\langle 1 \rangle}, \text{spmin}^{\langle 2 \rangle}, X\right)$

$Yr2 := \text{Milling\_Strain}^{\langle 3 \rangle}$ $\quad SB := \text{Milling\_Strain}^{\langle 4 \rangle}$ $\quad RSB := \text{rows}(SB) = 1303$

$\max(SB) = 27$ $\quad \min(SB) = -22$

$\text{rows}(\text{extrema}(SB)) = 946$ $\quad HHTB := \text{eemd}(SB, 0, 1)$ $\quad \text{cols}(HHTB) = 11$ $\quad i := 1..\text{rows}(HHTB)$

$r2 := 1..RSB$ $\quad XB_r := r2$ $\quad \text{ResidualB} := HHTB^{\langle 6 \rangle} + HHTB^{\langle 7 \rangle} + HHTB^{\langle 8 \rangle} + HHTB^{\langle 9 \rangle} + HHTB^{\langle 10 \rangle} + HHTB^{\langle 11 \rangle}$

$\text{Ext2} := \text{extrema}(SB)$ $\quad \text{RExt2} := \text{rows}(\text{Ext2})$ $\quad \text{spmax2} := \text{submatrix}\left(\text{Ext2}, 1, \text{Ext2}_{\text{RExt2},2}, 1, 2\right)$ $\quad \text{Ext2}_{\text{RExt2}-1,2} = -1$

$\text{spmin2} := \text{submatrix}\left(\text{Ext2}, \text{Ext2}_{\text{RExt2},2} + 1, \text{RExt2} - 2, 1, 2\right)$ $\quad up2_{sp} := \text{cspline}\left(\text{spmax2}^{\langle 1 \rangle}, \text{spmax2}^{\langle 2 \rangle}\right)$

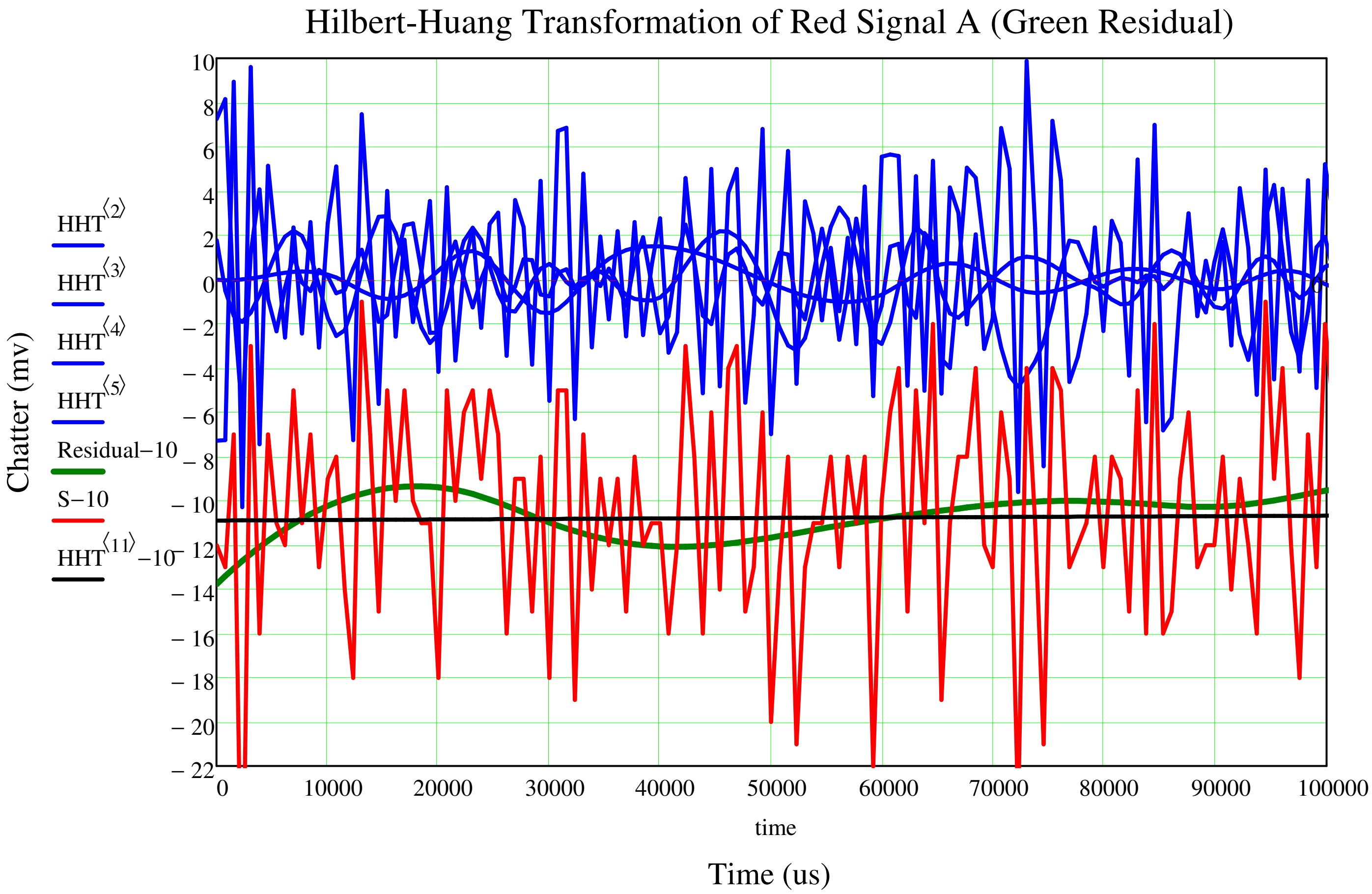
$\text{upper2} := \text{interp}\left(up2_{sp}, \text{spmax2}^{\langle 1 \rangle}, \text{spmax2}^{\langle 2 \rangle}, XB\right)$

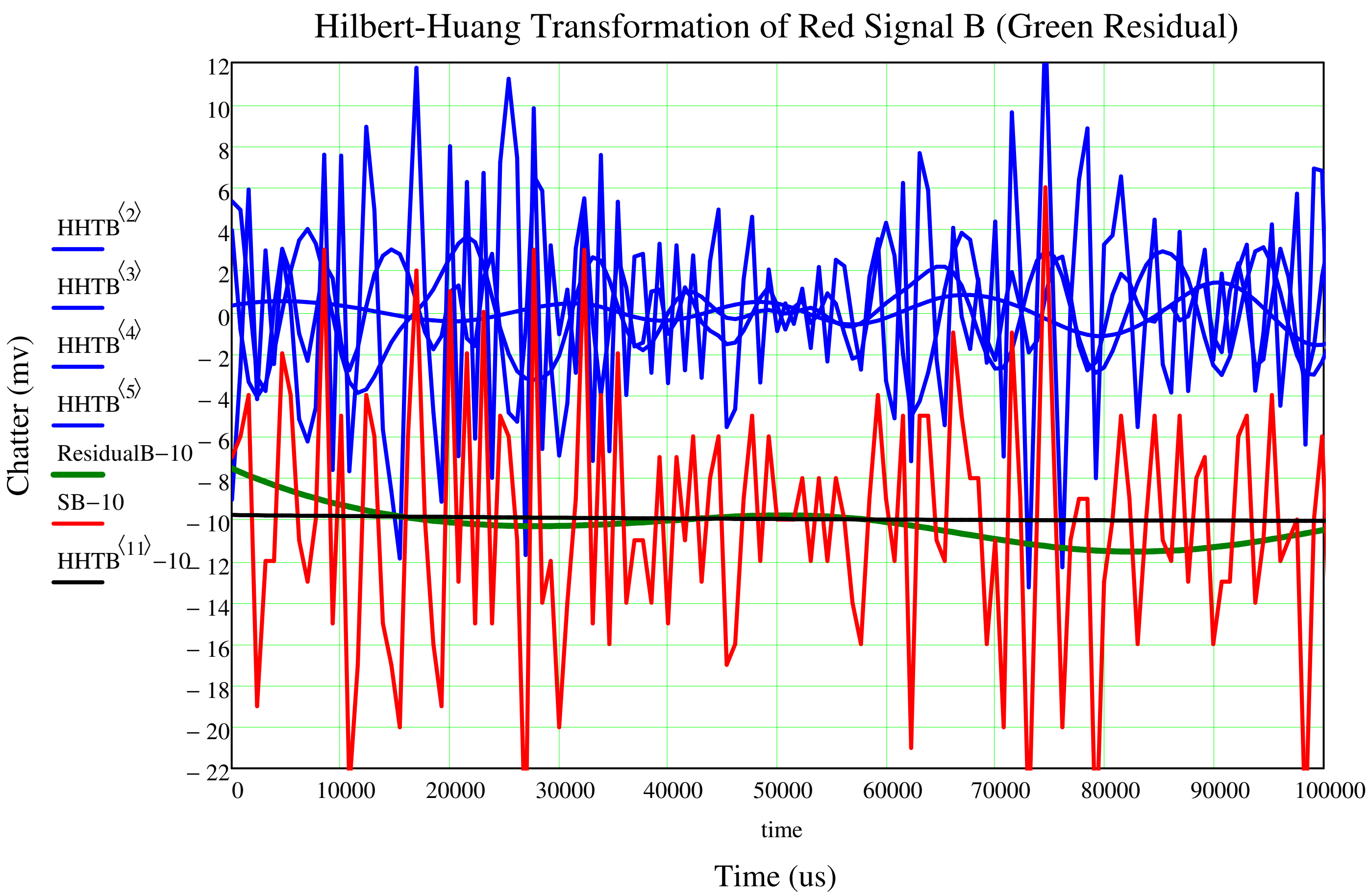
$low2_{sp} := \text{cspline}\left(\text{spmin2}^{\langle 1 \rangle}, \text{spmin2}^{\langle 2 \rangle}\right)$ $\quad \text{lower2} := \text{interp}\left(low2_{sp}, \text{spmin2}^{\langle 1 \rangle}, \text{spmin2}^{\langle 2 \rangle}, XB\right)$

**NOTE: The Array Origin of the worksheets is 1. HHT<1> is just the original signal.
Therefore HHT<5> is the Intrinsic Mode 4.**

Empirical Mode Decomposition of Milling Chatter

Note: Signal and Residual is offset by -10 units to display input signal separately





Ratio of Normalized Spectral Energy Content, NERatio

n starts with 2 so that the original signal is excluded from the Ratio

$$NERatio_n := \sum_{r=1}^{RT} \frac{(HHT_{r,n})^2}{\sum_{r=1}^{RT} (S_r)^2} \qquad NERatioB_n := \sum_{r=1}^{RT} \frac{(HHTB_{r,n})^2}{\sum_{r=1}^{RT} (SB_r)^2} \qquad \sum NERatio = 1.091 \qquad \sum NERatioB = 1.156$$

Comparison of Energy Ratios of Spectrum Content of B to A

$$ERatioBtoA_n := \frac{NERatioB_n}{NERatio_n}$$

Note: Sample B to A Normalized Energy Ratio of Intrinsic Mode 4 (HHT<5>, Z5) is 2.2.
See Graph of Z5 and ZB5 at bottom of next page.

Normalized Spectral Energy Content

$NERatio^T =$

	1	2	3	4	5	6	7	8	9	10
1	0	0.794	0.153	0.063	0.033	0.02	0.01	0.002	0.002	...

$NERatioB^T =$

	1	2	3	4	5	6	7	8	9
1	0	0.778	0.176	0.094	0.074	0.015	0.009	0.004	...

$ERatioBtoA^T =$

	1	2	3	4	5	6	7	8	9
1	0	0.98	1.153	1.485	2.235	0.757	0.915	1.856	...

Signal Analysis of Intrinsic Mode Functions, IMFs, from Empirical Mode Decomposition

Compute the transform for **Modes 1 to 5**: Build the complex signal: Find the discrete Fourier transform spectrum CFFT of the complex signals:

Data set A

$y2 := hilbert(HHT^{\langle 2 \rangle}) \qquad z2 := HHT^{\langle 2 \rangle} + j y2 \qquad Z2 := CFFT(z2)$

$y3 := hilbert(HHT^{\langle 3 \rangle}) \qquad z3 := HHT^{\langle 3 \rangle} + j y3 \qquad Z3 := CFFT(z3)$

$y4 := hilbert(HHT^{\langle 4 \rangle}) \qquad z4 := HHT^{\langle 4 \rangle} + j y4 \qquad Z4 := CFFT(z4)$

$y5 := hilbert(HHT^{\langle 5 \rangle}) \qquad z5 := HHT^{\langle 5 \rangle} + j y5 \qquad Z5 := CFFT(z5)$

$y6 := hilbert(HHT^{\langle 6 \rangle}) \qquad z6 := HHT^{\langle 6 \rangle} + j y6 \qquad Z6 := CFFT(z6)$

Data set B

$w2 := hilbert(HHTB^{\langle 2 \rangle}) \qquad zB2 := HHTB^{\langle 2 \rangle} + j w2 \qquad ZB2 := CFFT(zB2)$

$w3 := hilbert(HHTB^{\langle 3 \rangle}) \qquad zB3 := HHTB^{\langle 3 \rangle} + j w3 \qquad ZB3 := CFFT(zB3)$

$w4 := hilbert(HHTB^{\langle 4 \rangle}) \qquad zB4 := HHTB^{\langle 4 \rangle} + j w4 \qquad ZB4 := CFFT(zB4)$

$w5 := hilbert(HHTB^{\langle 5 \rangle}) \qquad zB5 := HHTB^{\langle 5 \rangle} + j w5 \qquad ZB5 := CFFT(zB5)$

$w6 := hilbert(HHTB^{\langle 6 \rangle}) \qquad zB6 := HHTB^{\langle 6 \rangle} + j w6 \qquad ZB6 := CFFT(zB6)$

Comparison of Frequency Spectrum of Vibration Samples A versus B
Sample B (See Plots ZB2 and ZB3) displays more chatter than sample A.