

ELECTRIC VEHICLE PERFORMANCE SIMULATION



GM Volt - Vehicle, Motor, Road, and Environmental Parameters:

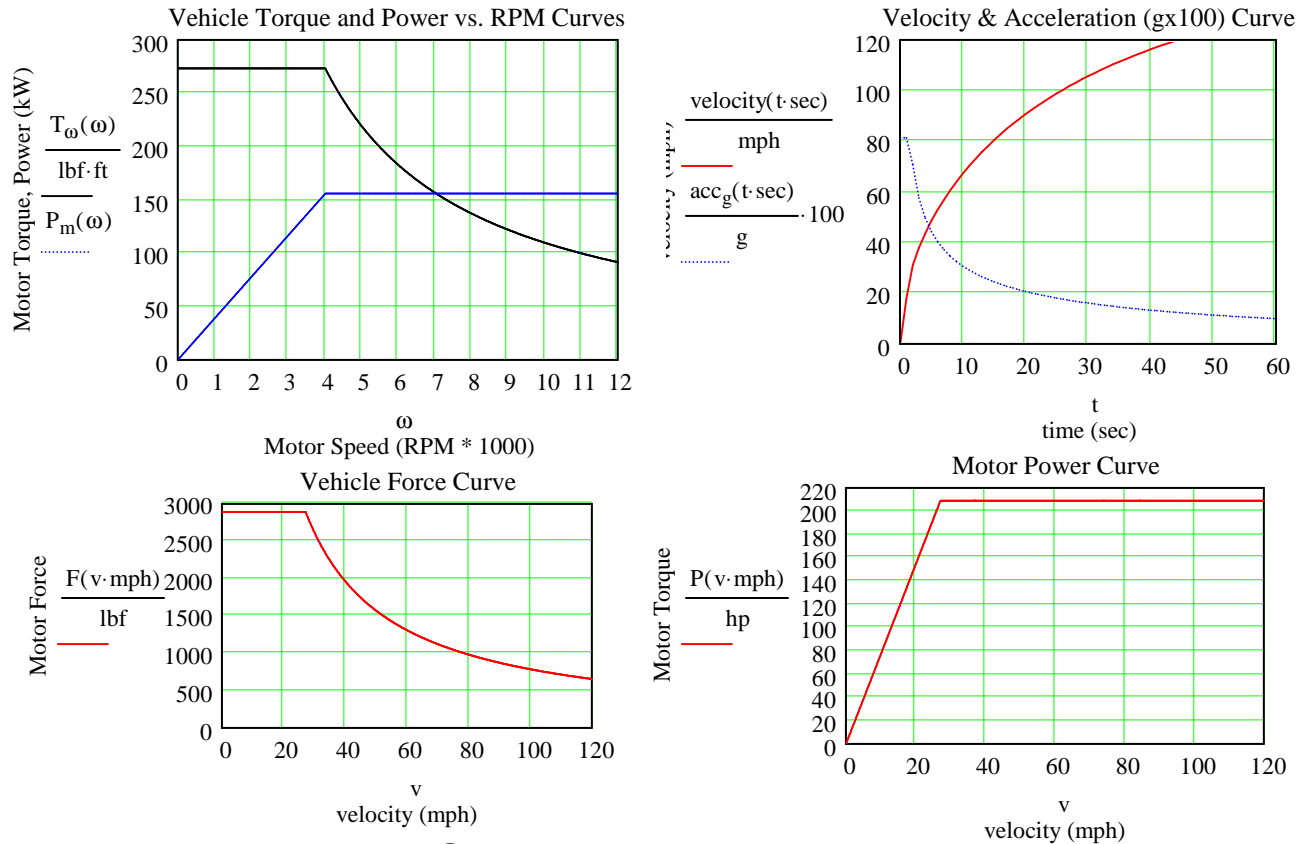
Max Motor Power:	$Power_{max} := 209 \cdot \text{hp}$	Gear Ratio ($v_{CP}=60\text{mph}$):	$GR := 12 \quad GR_2 := 8$
$P_{Generator} := 53 \cdot \text{kW}$	$Power_{max} := P_{Generator}$	Battery Energy:	$Energy_{bat} := 16 \cdot \text{kW} \cdot \text{hr}$
Max Motor Torque:	$T_m := 273 \cdot \text{ft} \cdot \text{lbf}$	Tire Radius*:	$r_{tire} := \frac{27.2}{2} \cdot \text{in} \quad 195/55R21$
Max Force, F_m	$F_m := GR \cdot \frac{T_m}{r_{tire}}$	$F_m = 2.891 \times 10^3 \text{ lbf}$	$RPM := \text{min}^{-1}$
$Power_{max} = 209 \text{ hp}$	$\omega_{CP} := \frac{Power_{max}}{T_m}$	$RPM_{CP} := \frac{\omega_{CP} \cdot \text{min}}{2 \cdot \pi}$	$\omega_{max} := 12000 \cdot \text{RPM}$
Constant Power Motor Torque, ω :	$v_{CP} := \frac{Power_{max}}{F_m}$	$v_{CP} = 27.114 \text{ mph}$	$RPM_{CP} = 4.021 \times 10^3$
Constant Power vehicle velocity, v_{CP} :	$t := 0, 1 \dots 61$	Average Wind Velocity:	$k := 10^3 \quad T_m = 370.138 \text{ N} \cdot \text{m}$ $V_w := 0 \cdot \text{mph}$
Time, in seconds:	$\tau := 1 \cdot \text{sec}$	Effective Cross Wind V:	$V_{cw} := 0 \cdot \text{mph}$
Time unit:	$SCF := 0.85$	Frontal Area*:	$A_{fg} := 2.16 \cdot \text{m}^2$
Shape Correction Factor:	$Cd := 0.245$	Frontal Area Corrected:	$A_f := A_{fg} \cdot SCF \quad Af = 1.836 \text{ m}^2$
Drag Coeff:	$Cd_{cw} := 0.000014$	Rolling Resistance Per Tire:	$RR_{tire} := 0.01$
Cross Wind Drag Coff:	$\rho := 1.3 \cdot \frac{\text{gm} \cdot \text{m}}{\text{liter} \cdot \text{er}}$	Tire Hysteresis, θ :	$\theta := \text{atan}(0)$
Air Density:	$RR_{road} := 0.0011 \cdot 0$	θ (radians):	(Average 0% road grade)
Road Rolling Resist:	$k_m := 1.06$	Curb Weight:	$M_{curb} := 3140 \cdot \text{lb}$
Rotational Inertia Coeff:	$M_{gross} := M_{curb} + \text{Passengers2}$	Passenger Weight:	$\text{Passengers2} := 170 \cdot \text{lb}$
Gross Weight:	$M_{gross} = 3.31 \times 10^3 \text{ lb}$	$M_{batt} := 300 \text{ lb}$	$MotorBrake_g = 0.824$
Motor Breaking Force in g:	$MotorBrake_g := GR \cdot T_m \cdot (k_m \cdot M_{gross} \cdot r_{tire} \cdot g)^{-1}$		

WeightToHP := $\frac{M_{curb}}{Power_{max}}$	<u>Vehicle Dynamics Equations:</u>	$WeightToHP = 15.024 \frac{\text{lb}}{\text{hp}}$
Velocity at Torque Fall:	$v_{maxT} := \omega_{max} \cdot 2 \cdot \pi \cdot r_{tire} \cdot GR_2^{-1}$	$v_{maxT} = 121.38 \text{ mph} \quad Th := 0 \cdot \text{sec} \cdot \text{mi}^{-1}$
Road Resistance, Ft:	$F_t(v) := M_{gross} \cdot g \cdot [Th \cdot v \cdot \sin(\theta) + (RR_{tire} + RR_{road}) \cdot \cos(\theta) + \sin(\theta)]$	
Aerodynamic Loss, Fa:	$F_a(v) := 0.5 \cdot \rho \cdot A_f \cdot [(v + V_w)^2 \cdot Cd + Cd_{cw} \cdot (0.5 \cdot v + V_{cw})^2]$	
Opposing Force, Fo:	$F_o(v) := F_a(v) + F_t(v)$	$F_o(60 \cdot \text{mph}) = 80.135 \text{ lbf}$
Ttractive Force:	$F(v) := \text{if} \left(v \leq v_{CP}, F_m, \frac{Power_{max}}{v} \right)$	$T(v) := F(v) \cdot \frac{r_{tire}}{GR}$
Third Law of Motion:	$a(v) := \frac{F(v) - F_o(v)}{k_m \cdot M_{gross}}$	$T_{\omega}(\omega) := T(\omega \cdot k \cdot 2 \cdot \pi \cdot r_{tire} \cdot GR^{-1} \cdot \text{RPM})$
(a is acceleration)		$P(v) := F(v) \cdot v$
		$P(60 \cdot \text{mph}) = 209 \text{ hp}$

Applying maximum motor torque, find the velocity starting from initial velocity = 0 mph.

Time := 0·sec	$velocity(t) := \text{root} \left(V - \int_0^t a(V) \cdot \tau \, dt, V \right)$	$P_m(\omega) := T_{\omega}(\omega) \cdot k \cdot 2 \cdot \pi \cdot \omega \cdot \text{RPM} \cdot \text{kW}^{-1}$
V := 0·mph		$velocity(60 \cdot \text{sec}) = 130.228 \text{ mph}$
	$time(v) := \text{root}(v - velocity(\text{Time}), \text{Time})$	$acc_g(t) := a(velocity(t \cdot \text{sec}))$
		time(60 mph) = 7.827s

GM VOLT PERFORMANCE SIMULATION CURVES:



Find the Single Charge (@SOC = 50%) Cruise Range for a given velocity

Driving Pattern/Profile:

Assume we cruise at constant speed and start, stop, and regen breaking every 15 minutes.

Drive Train Power Efficiency - Battery Loss to Force Commanded Vehicle Velocity:

State of Charge for generator is SOC_{gen} . **SOC_{gen} is 50% for recharge.** 320V HV battery idle power is P_o . 12V battery gives Accessory Power. The Traction Inverter x motor Efficiency - $TInvE$, HV Power Electronics at Idle Efficiency - $IPEE$, and Gear Power Efficiency - GPE are 90%, 95%, and 97%, respectively. Brake Regen efficiency of kinetic energy is 69% @ deceleration = 0.315g. Then the number of starts per hour as a function of velocity, NS , $NumStarts(v, P_o)$, is

$$TInvE := 0.90 \quad IPEE := 0.95 \quad GPE := 0.97 \quad Regen := 0.69 \quad v := 0, 2.. 80 \quad SOC_{gen} := 0.5$$

$$Power_{dissLoss}(v, P_o) := \frac{F_o(v) \cdot v}{TInvE \cdot GPE} + \frac{P_o \cdot watt}{IPEE}$$

$$RTEff := 0.92$$

$$Energy_{accel}(v) := Power_{max} \cdot time(v)$$

NS_o and NS are iterative converging estimates of $NumStarts$

$$NS_o(v) := 2 \cdot \left(\frac{50 \cdot mph}{v} \right)^2$$

$$NS(v, P_o, S) := \frac{Energy_{bat} \cdot (1 - S) - NS_o(v) \cdot \left(\frac{Energy_{accel}(v)}{TInvE \cdot GPE} - \frac{Regen \cdot M_{gross} \cdot v^2}{2} \right)}{Power_{dissLoss}(v, P_o) \cdot 15 \cdot min}$$

$$NumStarts(v, P_o, S) := floor \left[\frac{Energy_{bat} \cdot (1 - S) - NS(v, P_o, S) \cdot \left(\frac{Energy_{accel}(v)}{TInvE \cdot GPE} - \frac{Regen \cdot M_{gross} \cdot v^2}{2} \right)}{Power_{dissLoss}(v, P_o) \cdot 15 \cdot min} \right]$$

$$Cruise_Range(v, P_o, S) := \frac{Energy_{bat} \cdot (1 - S) - NumStarts(v, P_o, S) \cdot \left(\frac{Energy_{accel}(v)}{TInvE \cdot GPE} - \frac{Regen \cdot M_{gross} \cdot v^2}{2} \right)}{Power_{dissLoss}(v, P_o)} \cdot v$$

Single Charge Highway Cruise Range Estimate

$$\text{Cruise_Range}(30\text{-mph}, 50, \text{SOC}_{\text{gen}}) = 71.188 \text{ mi}$$

$$\text{Cruise_Range}(40\text{-mph}, 50, \text{SOC}_{\text{gen}}) = 59.5 \text{ mi}$$

$$\text{Cruise_Range}(50\text{-mph}, 50, \text{SOC}_{\text{gen}}) = 49.211 \text{ mi}$$

$$\text{Cruise_Range}(55\text{-mph}, 50, \text{SOC}_{\text{gen}}) = 43.835 \text{ mi}$$

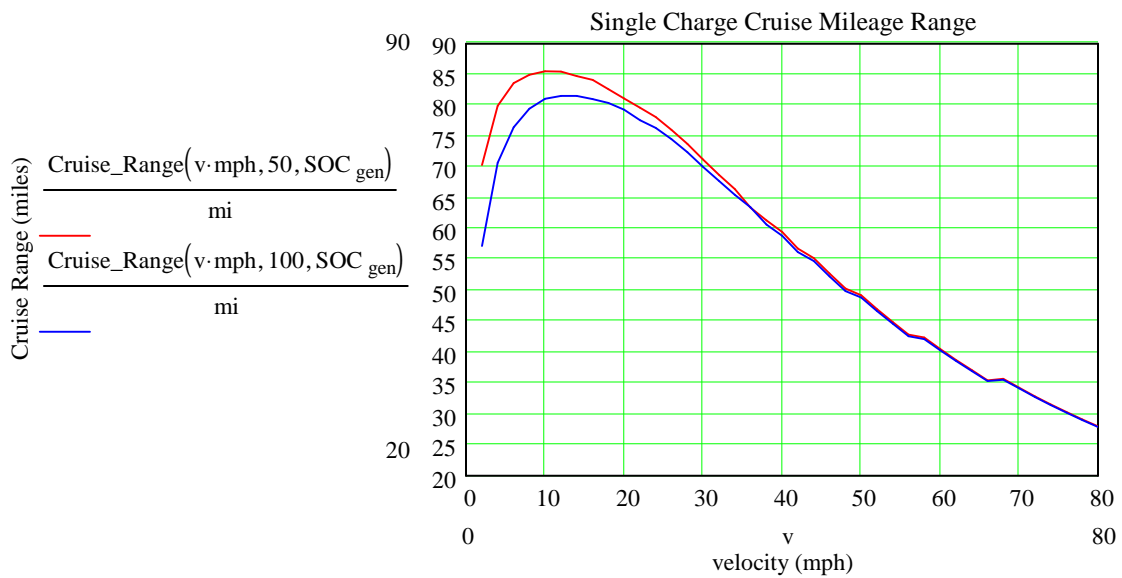
$$\text{Cruise_Range}(60\text{-mph}, 50, \text{SOC}_{\text{gen}}) = 40.515 \text{ mi}$$

$$\text{Cruise_Range}(70\text{-mph}, 50, \text{SOC}_{\text{gen}}) = 34.229 \text{ mi}$$

$$\text{Cruise_Range}(90\text{-mph}, 50, \text{SOC}_{\text{gen}}) = 22.892 \text{ mi}$$

$$\text{Cruise_Range}(100\text{-mph}, 50, \text{SOC}_{\text{gen}}) = 21.418 \text{ mi}$$

$$\text{Cruise_Range}(120\text{-mph}, 50, \text{SOC}_{\text{gen}}) = 15.861 \text{ mi}$$



Specsmanship: Twice as much range at 30 mph than 70 mph.

Conclusion: I need a bigger or a better battery!

Cruise Range as a Function of Traction Battery Idle Power, P_o

Cruise_Range(15·mph, 50, 0.3) = 118.09 mi

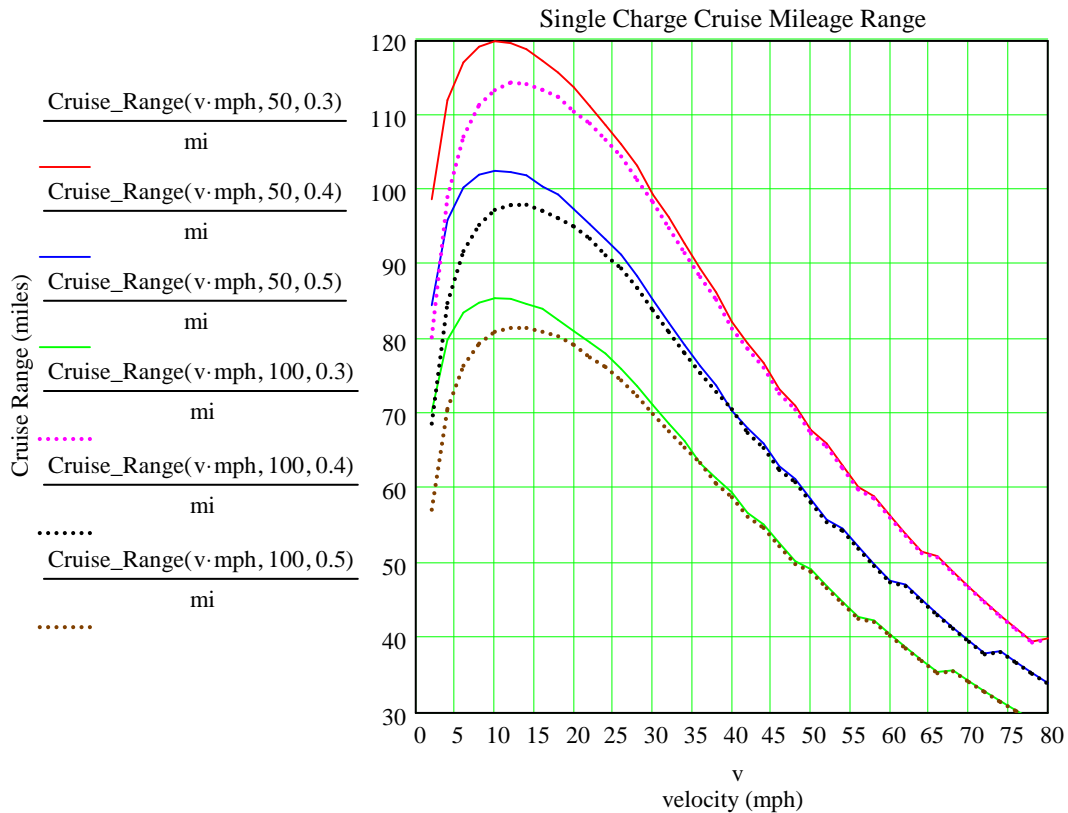
$v := 0, 2.. 80$

Cruise_Range(55·mph, 50, 0.5) = 43.835 mi

$180 \cdot \frac{\text{km}}{\text{hr}} = 111.847 \frac{\text{mi}}{\text{hr}}$

$\frac{0.5 - 0.25}{0.5} = 0.5$

$\frac{\text{Cruise_Range}(55 \cdot \text{mph}, 100, 0.25) - \text{Cruise_Range}(55 \cdot \text{mph}, 100, 0.5)}{\text{Cruise_Range}(55 \cdot \text{mph}, 100, 0.5)} = 0.516$



A123 Systems NanoPhosphate m1 Cell Characterization / GM Volt Battery:

$\text{SpecificEnergy}_{A123m1} := \frac{3.3 \cdot \text{volt} \cdot 2.3 \cdot \text{amp} \cdot \text{hr}}{70 \cdot \text{gm}}$

$\text{SpecificEnergy}_{A123m1} = 108.429 \frac{\text{watt} \cdot \text{hr}}{\text{kg}}$

Weight Aux: Case, Blower, Cooler, Relays, Electronics $\text{Weight}_{\text{Aux}} := 25 \cdot \text{lb}$ $\text{Weight}_{\text{bat}} := 350 \cdot \text{lb}$

$\text{Energy}_{A123\text{Bat}} := \text{Weight}_{\text{bat}} \cdot \text{SpecificEnergy}_{A123m1}$

Energy_{A123Bat} = 17.214 k·watt·hr

$\text{SpecificEnergy}_{\text{GoalBat}} := \frac{150 \cdot \text{k} \cdot \text{watt} \cdot \text{hr}}{\text{kg}}$

$\text{Energy}_{\text{Goal}} := \text{Weight}_{\text{bat}} \cdot \text{SpecificEnergy}_{\text{GoalBat}}$

Energy_{Goal} = 2.381 × 10⁴ k·watt·hr

Series Parallel Arrangement of A123 Systems ANR26650M1 Cells:

$$\text{NumberCells} := \frac{\text{Weight}_{\text{bat}}}{70 \cdot \text{gm}} \quad \text{NumberCells} = 2.268 \times 10^3 \quad \text{SeriesCells} := \frac{363 \cdot \text{volt}}{3.3 \cdot \text{volt}} \quad \text{SeriesCells} = 110$$

$$\text{ParallelCells} := \frac{\text{NumberCells}}{\text{SeriesCells}} \quad \text{ParallelCells} = 20.618 \quad \text{ContinuousDischargeCell} := 70 \cdot \text{amp}$$

$$\text{ContinuousDischargeCell} \cdot 20 = 1.4 \times 10^3 \text{ A} \quad \text{ContinuousPower} := 363 \cdot \text{volt} \cdot \text{ContinuousDischargeCell} \cdot 20$$

$$\text{ContinuousPower} = 508.2 \text{ k-watt} \quad \text{Capacity} := 2.3 \cdot \text{A} \cdot \text{hr} \cdot 20 \cdot 363 \cdot \text{volt} \quad \text{Capacity} = 16.698 \text{ k-watt-hr}$$

$$\text{CostCellDollars} := \frac{110}{6} \quad \text{CostBattery} := \text{CostCellDollars} \cdot 20 \cdot 110 \quad \text{CostBattery} = 4.033 \times 10^4$$

Goals for 2010 USABC

$$\text{CostBatteryGoal2010} := \frac{500 \cdot \text{Capacity}}{\text{k-watt-hr}} \quad \text{CostBatteryGoal2010} = 8.349 \times 10^3$$

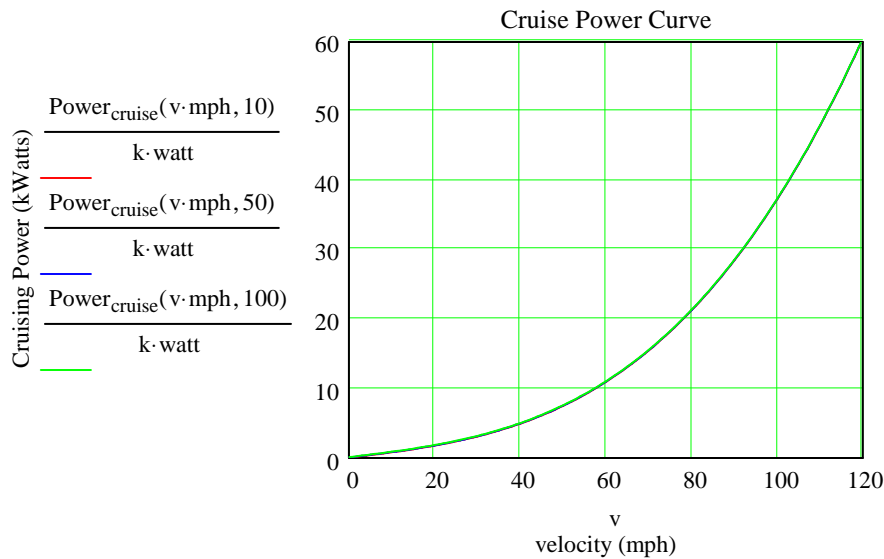
$$\text{PeakMotorCurrent} := \frac{\text{Power}_{\text{max}}}{363 \cdot \text{volt}} \quad \text{PeakMotorCurrent} = 429.342 \text{ A} \quad 16 \text{ k-watt}$$

$$\text{MotorStallCurrent} := \blacksquare$$

Find the Power to Maintain Constant Velocity

$$\text{Power}_{\text{cruise}}(v, P_o) := \text{Power}_{\text{dissLoss}}(v, P_o)$$

$$\text{Power}_{\text{cruise}}(60 \cdot \text{mph}, 100) = 11.057 \text{ kW} \quad v := 0, 1 \dots 120$$



Note: The Volt generator's output is 54 kW.
This allows it produce a net charge up to 80 mph cruise.

AER Given Three Different Driving Schedules

Read US06 and FTP Driving Profile Files

<http://www.epa.gov/nvfel/testing/dynamometer.htm>

The US06 cycle represents an 8.01 mile (12.8 km) route with an average speed of 48.4 miles/h (77.9 km/h), maximum speed 80.3 miles/h (129.2 km/h), and a duration of 596 seconds.

The Federal Test Procedure(FTP) is composed of the UDDS followed by the first 505 seconds of the UDDS. It is often called the EPA75. FP10 is a 10 Hz Sampling. HY10 is the 10 Hz Highway schedule.

```

FTPF := READPRN("FedTestProc.txt")      t := FTPF<0>      FTP := FTPF<1>      rows(FTP) = 1.875 × 103
UDDSF := READPRN("uddscol.txt")          UDDS := UDDSF<1>  rows(UDDS) = 1.37 × 103
HWYF := READPRN("hwycol.txt")           HWY := HWYF<1>   Rhwy := rows(HWY)
FP10 := READPRN("FTP10Hz.TXT")           FTP10V := submatrix(FP10,0,rows(FP10) - 1,1,cols(FP10) - 1)
HY10 := READPRN("HWY10Hz.TXT")          HWY10V := submatrix(HY10,0,rows(HY10) - 1,1,cols(HY10) - 1)
US06F := READPRN("US06PROFILE.TXT")     time := US06F<0>  US06 := US06F<1>      n6 := 0..598
    
```

Calculate All Electric Range, AER, for Driving Profile Velocity/Time File, P and Sampling Rate, Hz

Regen Efficiency Curve vs Decel (g): $REff(g) := \frac{85}{77} \cdot 0.01 \cdot \left[\left(1 - e^{-27.129 \cdot g} \right) \cdot 91.235 - 28.408 \right]$ $Gg := \frac{mph}{sec \cdot g}$

```

AER(P, Hz) := | Ebat ← Ediss ← vold ← 0
                | n ← -1
                | N ← rows(P) - 1
                | while Ediss < 8 ∧ n = n
                |   | n ← n + 1
                |   | t ← mod(n, N)
                |   | v ← Pt
                |   | vavg ← (v + vold) · 0.5
                |   | Paccel ←  $\frac{k_m \cdot M_{gross} \cdot (v - v_{old}) \cdot \frac{mph \cdot Hz}{sec} \cdot v_{avg} \cdot mph}{T_{InvE} \cdot GPE}$  if v > vold
                |   | Paccel ←  $k_m \cdot M_{gross} \cdot (v - v_{old}) \cdot \frac{mph \cdot Hz}{sec} \cdot v_{avg} \cdot mph \cdot REff\left[\frac{(v_{old} - v) \cdot Hz \cdot Gg}{1}\right]$  otherwise
                |   | Ediss ← Ediss +  $\frac{(Power_{dissLoss}(v \cdot mph, 100) + P_{accel}) \cdot sec}{kW \cdot hr \cdot Hz}$ 
                |   | vold ← v
                |   | Ebatn ← Ediss
                |   | R ←  $\sum_{m=0}^n \frac{(P_{mod(m, N)} + P_{mod(m+1, N)}) \cdot mph \cdot sec}{2 \cdot mi \cdot Hz}$ 
                | R
    
```

$$r1 := 0..rows(HY10) \cdot 10 - 1 \quad HWY10_{r1} := HWY10V_{\text{ceil}\left(\frac{r1+1}{10}\right)-1, \text{mod}(r1, 10)}$$

$$AER(US06, 1) = \blacksquare \quad AER(FTP, 1) = 40.138 \quad AER(HWY, 1) = 40.095 \quad AER(HWY10, 10) = \blacksquare$$

EPA 20085 Cycle MPG Fuel Economy Least Squares Fit Regression for AER

$$MPG_{\text{city}} := \frac{1}{\left(0.003259 + \frac{1.18053}{AER(FTP, 1)}\right)} \quad MPG_{\text{city}} = 30.608 \quad MPG_{\text{hwy}} := \frac{1}{0.001376 + \frac{1.3466}{AER(HWY, 1)}} \quad X := \frac{1}{40}$$

$$MPG_{\text{epa}} := 0.55 \cdot MPG_{\text{city}} + 0.45 \cdot MPG_{\text{hwy}} \quad MPG_{\text{epa}} = 29.706$$

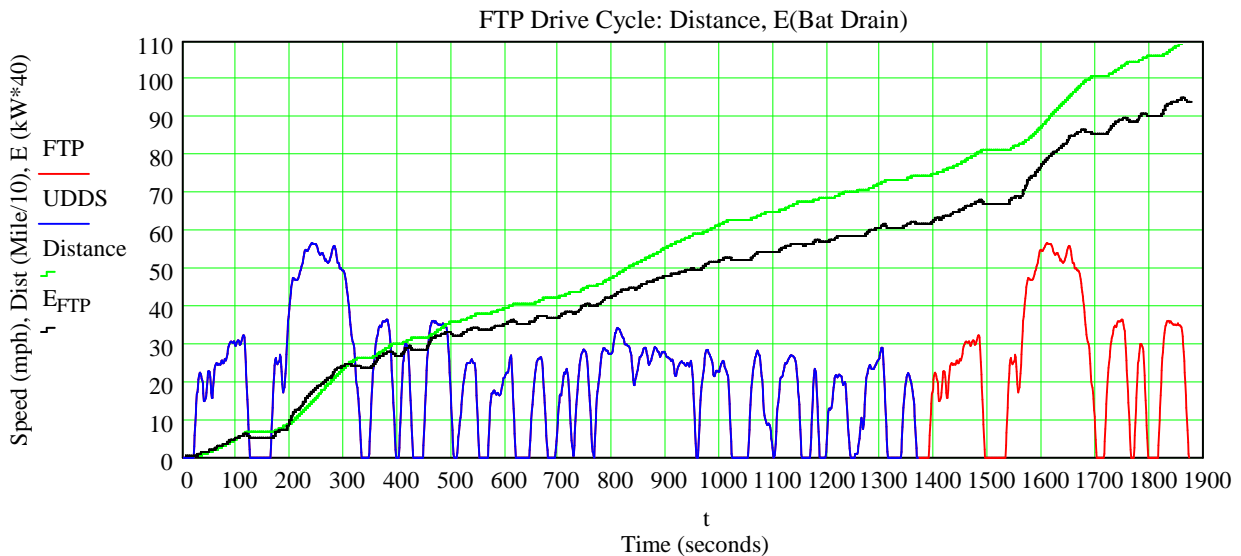
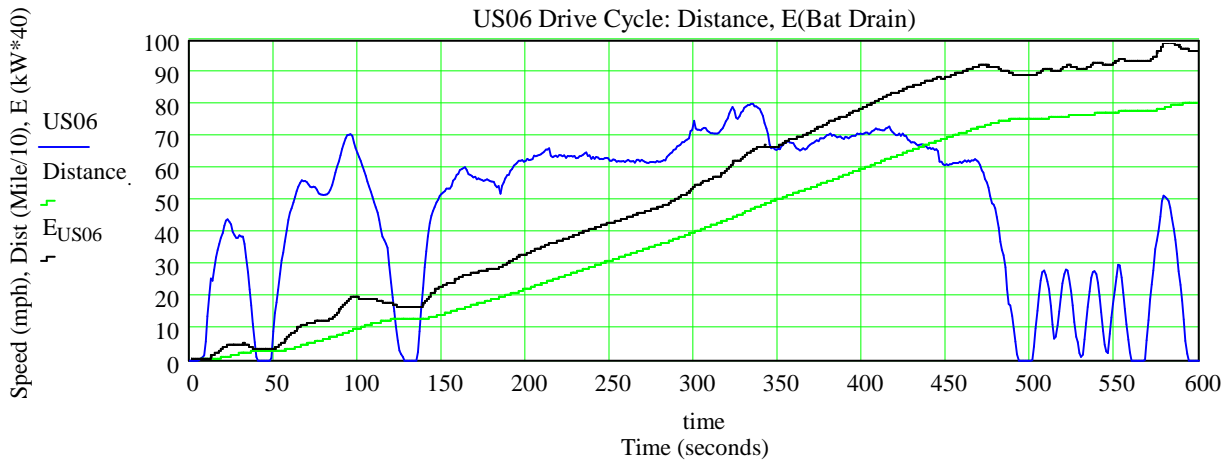
$$r := 0..rows(FTP) - 1 \quad \text{Distance}_r := \sum_{r=0}^r FTP_r \cdot \frac{10}{60 \cdot 60} \quad rr := 0..rows(US06) - 1 \quad \text{Distance}_{rr} := \sum_{rr=0}^{rr} US06_{rr} \cdot \frac{10}{60 \cdot 60}$$

$$\max(\text{Distance}) = 110.414 \quad \max(\text{Distance}_{rr}) = 80.08$$

$$\text{WRITEPRN}("EFTP.PRN") := AER(FTP, 1) \cdot 40 \quad E_{FTP} := \text{READPRN}("EFTP.PRN") \quad \max(E_{FTP}) \cdot X = 2.369$$

$$\text{WRITEPRN}("EUS06.PRN") := AER(US06, 1) \cdot 40 \quad E_{US06} := \text{READPRN}("EUS06.PRN") \quad \max(E_{US06}) \cdot X = 2.487$$

$$\text{WRITEPRN}("EHWY.PRN") := AER(HWY, 1) \cdot 40 \quad E_{HWY} := \text{READPRN}("EHWY.PRN") \quad \max(E_{HWY}) \cdot X = 40.1$$



Profile: Max Acceleration, ads, Max Power Dissipation, Pds, Estimate of AER (#Charges x Distance)

$$\begin{aligned}
 a_{ds_{n_6}} &:= \left(US06_{n_6+1} - US06_{n_6} \right) \cdot \text{mph} \cdot (1 \cdot \text{sec})^{-1} & P_{ds_{n_6}} &:= k_m \cdot M_{\text{gross}} \cdot a_{ds_{n_6}} \cdot US06_{n_6} \cdot \text{mph} \\
 \min(a_{ds}) &= -0.315 \text{ g} & n_h &:= 0.. R_{\text{hwy}} - 2 & \max(P_{ds}) &= 94.045 \text{ hp} & \max(US06) &= 80.3 \\
 REff(0.315) &= 0.693 \\
 a_{ds_{n_h}} &:= \left(HWY_{n_h+1} - HWY_{n_h} \right) \cdot \text{mph} \cdot (1 \cdot \text{sec})^{-1} & P_{ds_{n_h}} &:= k_m \cdot M_{\text{gross}} \cdot a_{ds_{n_h}} \cdot HWY_{n_h} \cdot \text{mph} \\
 \max(a_{ds}) &= 0.146 \text{ g} & \frac{8 \cdot 11.04 \cdot 40}{\max(E_{\text{FTP}})} &= 37.278 & \max(P_{ds}) &= 31.34 \text{ hp} & \frac{8 \cdot 8.008 \cdot 40}{\max(E_{\text{US06}})} &= 25.762 \\
 \sum_{r=0}^{1380} \text{FTP}_r \cdot \frac{1}{60 \cdot 60} &= 7.45 & E_{\text{FTP}_{1380}} &= 61.72 & \frac{8 \cdot 7.45 \cdot 40}{E_{\text{FTP}_{1380}}} &= 38.626
 \end{aligned}$$

PayOff Battery Lease

$$\begin{aligned}
 M &:= 8000 & \text{num} &:= 12 & \text{term} &:= 4 & \text{nper} &:= \text{num} \cdot \text{term} & R_M &:= 0.05 \\
 \text{PMT}_M &:= \text{pmt} \left(\frac{R_M}{\text{num}}, \text{nper}, M \right) & \text{PMT}_M &= -184.234 & \text{PMT}_M \cdot 12 &= -2.211 \times 10^3
 \end{aligned}$$

Cost of Gas vs. Electric for Months Driving

$$\begin{aligned}
 \text{Miles_Yr} &:= 12000 \cdot \text{mi} & \text{GasCost_Month} &:= \frac{\text{Miles_Yr}}{20 \cdot \text{mi}} \cdot \frac{4.2}{12} & \text{GasCost_Month} &= 210 \\
 \text{CostDollar}_{\text{elect}} &:= \frac{0.1}{\text{kW} \cdot \text{hr}} & \text{charge} &:= \frac{8 \cdot \text{kW} \cdot \text{hr}}{40 \cdot \text{mi}} & \text{Cost_mile}_{\text{elect}} &:= \text{CostDollar}_{\text{elect}} \cdot \text{charge} \\
 \text{CostDollarsMonthEV}_{\text{elect}} &:= \text{Cost_mile}_{\text{elect}} \cdot \frac{\text{Miles_Yr}}{12} & \text{CostDollarsMonthEV}_{\text{elect}} &= 20 \\
 \text{LeasePlusElect_Month} &:= -\text{PMT}_M + \text{CostDollarsMonthEV}_{\text{elect}} & \text{LeasePlusElect_Month} &= 204.234 \\
 \text{GasCost_Month} &= 210 & \text{charge} &= 447.387 \text{ kg m s}^{-2} \\
 & & \frac{1000 \cdot \text{mi}}{\text{charge}} &= 3.597 \times 10^3 \text{ s}^2 \text{ kg}^{-1} \\
 \frac{\text{Miles_Yr}}{20 \cdot \text{mi}} \cdot \frac{\text{CostGas}}{12} &= 299 + 24 & 8 \cdot \text{kW} \cdot \text{hr} \cdot \text{CostDollar}_{\text{elect}} \cdot 30 &= 24
 \end{aligned}$$

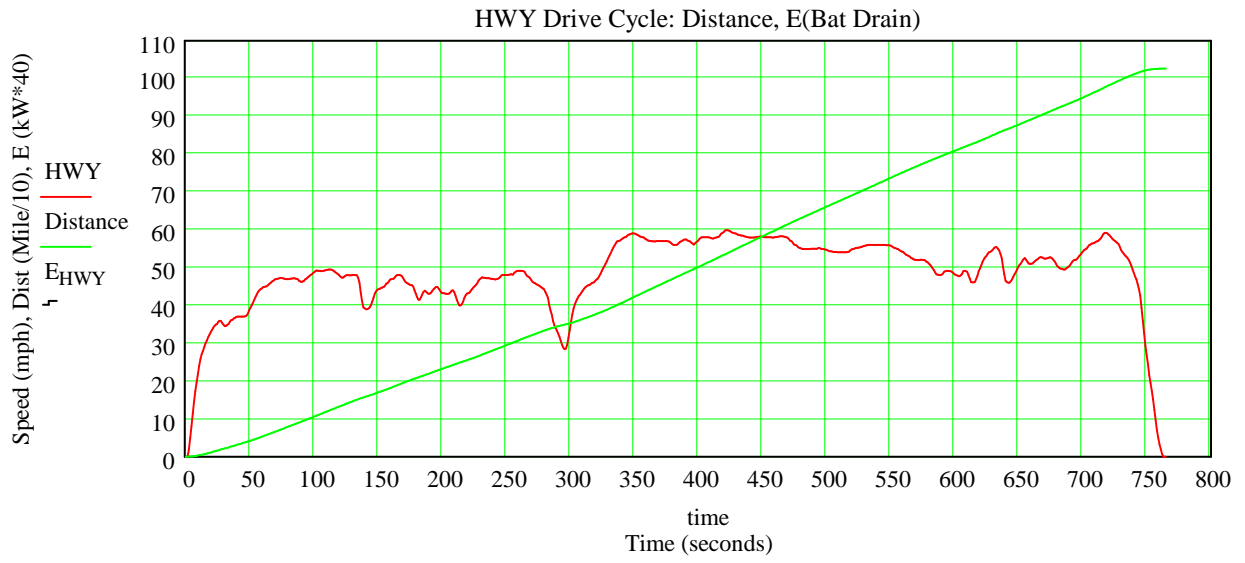
r := 0..rows(HWY) - 1

$$\text{Distance}_r := \sum_{r=0}^r \text{HWY}_r \cdot \frac{10}{60 \cdot 60}$$

time := HWYF⁽⁰⁾

mean(HWY) = 48.204

max(HWY) = 59.9



$$D(SW, AR, RD) := \frac{SW \cdot AR \cdot 2}{25.4 \cdot 100} + RD$$

$$D(195, 55, 21) = 29.445$$

$$D := 0 \cdot \text{mi}$$

$$\text{QMile}(d) := \text{root} \left(d - \text{root} \left(D - \int_0^{\text{Time}} \text{velocity}(\text{Time}) \cdot \tau \, d\text{Time}, D \right), \text{Time} \right)$$

$$\text{QMile}(0.25 \cdot \text{mi}) = \blacksquare \quad 23 \text{ sec}$$

$$0.95 \cdot 0.95 = 0.903$$

