Top Fuel Dragster Performance Simulation

The Simulation can be run or modified with Mathcad 14/15. Free Trial at: **http://www.ptc.com/product/mathcad/free-trial**

Mathcad Simulation at: http://www.LeapCad.com/Top Dragster Performance Simulation.xmcd 9-19-2015

VXPhysics.com

Goal: Simulate Top Fuel Dragster Performance

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I. Introduction - Basics

Examining the 2010 Kalitta Motorsports Dragster Specs (2014 has 1.25X Power & Torque)

Specifications March 13, 2010
http://www.popularmechanics.com/cars/a12596/anatomy-of-a-top-fuel-dragster/

Max hp: 8,000 hp (2014 Specs 10,000 hp) **1.25X**
Torque: 8100 N-M, 6000 lbf ft (2014 Specs 7400 ft lbf) **1.23X**
90% nitromethane 10% alchohol, 5 gal/4 sec, with 700 hp supercharger
Clutch is Critical: 5 disc clutch - no transmission. Controlled by centrepital force.
Gear Ratio: 3.20 to 1
Goodyear large diameter, large width,low inflation pressure tire.
R36x17.5-16" 36" diameter at start, expand to 44 in, average 40 in
Track coated with adhesive for maximum traction.
Tire Coefficient of Friction: 4
Front wheels are off the ground for first 200 ft of run
Magnetos 44 amps per spark plug
Weight 2300 lbf

Performance Specs, Perf Table

0.5 s	10.27 ft	73.89 mph	
1sec	52 ft	113.82 mph	
1.5s	125 ft	162 mph	
2 s	232 ft	213 mph 7	7200 rpm
2.5 s	379.9 ft	248.5 mph	6599 rpm
3 s	566 ft	271.6 mph	
3.83 s	1000 ft	321.6 mph	

Performance Table: sec, ft, mph

$$\mathrm{PerfTab} := \begin{pmatrix} 0.05 & 0 & 4 \\ 0.5 & 10.27 & 73.89 \\ 1 & 52 & 113.82 \\ 1.5 & 125 & 162 \\ 2 & 232 & 213 \\ 2.5 & 379.9 & 248.5 \\ 3 & 566 & 271.6 \\ 3.83 & 1000 & 321.6 \end{pmatrix}$$

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II. Macro Performance Model Discussion & Description of the Model

Macro Model: Macro Models requires only limited knowledge of internal parameters. We treat the system as a Black Box. That is, we don't know the details of what's inside, just a few fundamental parameters. We are only interested in overall performance. Ignore the intricacies. Simple, but not too simple. May not know what is inside, but regardless, the laws of Physics still apply. **This model ignores the initial boost in acceleration from aerodynamic down forces and the 700 hp Power Losses needed to operate the SuperCharger. It does include aerodynamic and tire drag forces.**
We will just use the basic physical parameters such as given in Section III and Section IV Models:

Read Data Plot Data for Power and Torque vs. RPM 2009 Castillo

PowerPlot := READPRN("Castillo Top Drag RPM vs Power.csv") − 12 rows(PowerPlot) = 56

TorquePlot := READPRN("Castillo Top Drag RPM vs Torque.csv") rows(TorquePlot) = 54

Fit Power Curves to Data Plots

Guess $a := 1 \quad b := 1 \quad c := 1 \quad d := 27 \quad e := 1 \quad f := 30$

Given $\mathrm{Torq} := \mathrm{TorquePlot}^{\langle 1\rangle} \qquad \mathrm{spd} := \mathrm{TorquePlot}^{\langle 0\rangle}$

$$\mathrm{Torq} - \left[a\cdot(\mathrm{spd})^3 - b\cdot(\mathrm{spd}-c)^2 - d\cdot(\mathrm{spd}-e) + f\right] = 0$$

$$\mathrm{At} := \mathrm{Minerr}(a,b,c,d,e,f)$$

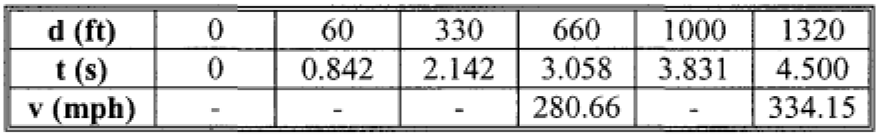
$$\mathrm{Torque}(s) := \mathrm{At}_0\cdot(s)^3 - \mathrm{At}_1\cdot\left(s-\mathrm{At}_2\right)^2 - \mathrm{At}_3\cdot\left(s-\mathrm{At}_4\right) + \mathrm{At}_5$$

$\mathrm{Torque}(6000) = 5844.32 \qquad S := 2000, 2100 .. 9000$

$\mathrm{Torque}_{max} := \max(\mathrm{Torq}) = 5799.13$

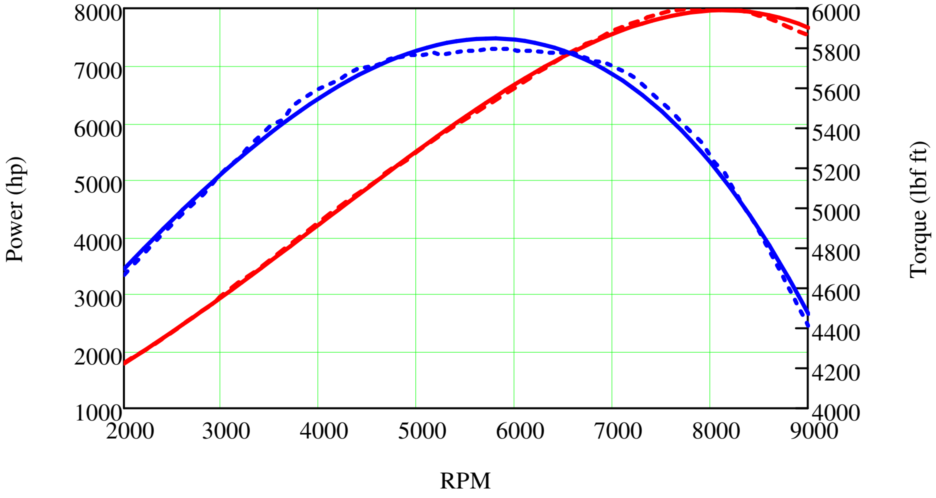
Calculate Power as Torque x RPM

$$\mathrm{Power}(\mathrm{RPM}) := \mathrm{Torque}(\mathrm{RPM})\cdot\frac{\mathrm{RPM}\cdot\mathrm{lbf}\cdot\mathrm{ft}\cdot 2\cdot\pi}{60\cdot s}$$



d (ft)	0	60	330	660	1000	1320
t (s)	0	0.842	2.142	3.058	3.831	4.500
v (mph)	-	-	-	280.66	-	334.15

Top Dragster Power (Red-hp) and Torque(Blue-lbf ft) vs RPM



III. Specifications & Engineering Estimates: Peak Acceleration

Max Power: $\mathrm{Power_{max}} := 8000 \cdot \mathrm{hp}$ $\mathrm{RPM_{max}} := 9000$ Gear Ratio: $\mathrm{GR} := 3.20$

Max Torque $T_{max} := \mathrm{Torque_{max}} \cdot \mathrm{lbf} \cdot \mathrm{ft}$ Rear Tire: R36x17.5-16" $r_{tire} := \frac{40}{2} \mathrm{in}$ $F_{Motor\_Max} := \frac{T_{max} \cdot \mathrm{GR}}{r_{tire}}$

$T_{max} = 5799.13 \, \mathrm{lbf} \cdot \mathrm{ft}$

Tire Coefficient of Friction, μ: $\mu := 4$ $\mathrm{car}_{max\_g} := \mu \cdot g$ $k := 1000$ $\tau := 1 \cdot \mathrm{sec}$

Curb Weight: $M_{curb} := 2300 \mathrm{lbm}$ $M_{gross} := M_{curb} + 160 \mathrm{lbm} = 2460 \cdot \mathrm{lbm}$

Aerodynamic Drag Coeff (TM): $\mathrm{Cd} := 0.35$ Average Wind Velocity: $\mathrm{Vw} := 0 \cdot \mathrm{mph}$

Cross Wind Drag Coff: $\mathrm{Cd_{cw}} := 0.000014$ Effective Cross Wind V: $V_{cw} := 0 \cdot \mathrm{mph}$

Shape Correction Factor: $\mathrm{SCF} := 0.85$ Vehicle Frontal Dimensions: $\mathrm{Af} := (57 - 7.9) \mathrm{in} \cdot 77 \cdot \mathrm{in}$

Air Density, tire resistance: $\rho := 1.293 \cdot \frac{\mathrm{gm}}{\mathrm{liter}}$ Drag Frontal Area $\mathrm{Ad} := \mathrm{Af} \cdot \mathrm{SCF}$ $\mathrm{Ad} = 2.07 \cdot \mathrm{m}^2$

Tire Rolling Resist, Hys: $\mathrm{RR_{tire}} := 0.011$ $T_{hys} := 0 \cdot \frac{\mathrm{sec}}{\mathrm{m}}$

Road Rolling Resistance: $\mathrm{RR_{road}} := 0.007$

Effective Mass Coefficient: $k_m := 1.0447$ $g_{max} := \frac{T_{max} \cdot \mathrm{GR}}{M_{gross} \cdot k_m \cdot r_{tire} \cdot g}$ $g_{max} = 4.33$ $\mathrm{RPM_{motor}} := \frac{\mathrm{Power_{max}}}{T_{max} \cdot 2 \cdot \pi}$

Clutching - The Most Critical Element in Top Fuel Dragster Performance

In drag racing, the clutch acts as the buffer between the power produced by the engine and the tires trying to apply it to the track surface. These cars, especially the fuel cars, are making so much power that it can't possibly be put to the ground all at once. There has to be something to take that power and apply it progressively through the RPM range or through the run, and allow the car to launch from a dead stop with a percentage of the engine's power then apply 100% of it when possible.

IV. Macro Model of Motor Dynamics: Tire Velocity is v, ωk is RPM/1000

Clutch, Tire, Down-Force Traction Model: Perfect Clutching - Constant Torque

Simple Step Model of Tire Traction (Assume perfect weight distribution, aerodynamics & wing design--> aceleration ~ 4g)

Tires do not have perfect grip, they may slip. Vehicle acceleration, a_{veh} is limited to the μ = tire road force/vehicle weight tire traction (tire_{max_g}) ~ 4g. The tire rpm x GR = motor rpm, but because of slip, **initially**, tire velocity can be greater than vehicle velocity. When critical velocity is attained, the body and tire aerodynamics apply down force to increase μ > 4.

$\mathrm{RPM} \times r_{tire}/\mathrm{GR} > v_{tire} = v_{vehicle}$

Case #1 Model Assumptions

Perfect Clutching - Clutch Spin for max Torque.

Clutching to allow max motor torque to be applied to tires/road. Perfect Tire.
Acceleration initially jumps to max Torque, that is, to 4.3 g.

$\mathrm{RPM} \times r_{tire}/\mathrm{GR} = v_{tire} = v_{vehicle}$

Case #2 Model Assumptions

No Clutch or Tire Spin - Applied Torque & Power Follow the RPM Cu

Motor rpm has no slip to vehicle speed.
Acceleration rises, peaks at **4.3 g, and then falls**.

Performance Model **This model ignores various power losses and the boost in acceleration from aerodynamic down forc**

Angular Velocity Symbol, Ω (units of radians/second) $\Omega(\omega) := 2\pi 1000 \cdot \omega \cdot \mathrm{min}^{-1}$ RPM/1000 Symbol, ω_k $\mathrm{RPM} := \mathrm{min}^{-1}$

Angular Vel Ω @Max Power: $\Omega_{Pmax} := \mathrm{Power_{max}} \cdot T_{max}^{-1}$ $\mathrm{RPM_{Pmax}} := \frac{\Omega_{Pmax}}{2 \cdot \pi}$ $\mathrm{RPM_{Pmax}} = 7245.38 \cdot \mathrm{RPM}$

Convert velocity to RPM $\mathrm{VtoRPM}(v_v) := v_v \cdot (1000 \cdot 2 \cdot \pi \cdot r_{tire} \cdot \mathrm{RPM})^{-1}$ $\omega_{Pfall} := \mathrm{RPM_{Pmax}} \cdot k^{-1} = 7.25 \cdot \mathrm{RPM}$

Tire Velocity at Torque Fall: $v_{Tfall} := \mathrm{RPM_{Pmax}} \cdot 2 \cdot \pi \cdot r_{tire} \cdot \mathrm{GR}^{-1}$ $v_{Tfall} = 269.44 \cdot \mathrm{mph}$

Tire Velocity to kRPM: $\mathrm{VtokR}(v_t) := v_t \cdot (k \cdot 2 \cdot \pi \cdot r_{tire} \cdot \mathrm{RPM})^{-1}$ $\mathrm{VtokR}(60 \cdot \mathrm{mph}) \cdot \mathrm{GR} = 1.61$ $\theta := 0$

Road Resistance, Ft: $\mathrm{Ft}(v_v) := M_{gross} \cdot g \cdot \left[T_{hys} \cdot v_v \cdot \sin(\theta) + (\mathrm{RR_{tire}} + \mathrm{RR_{road}}) \cdot \cos(\theta) + \sin(\theta)\right]$ $\mathrm{RPM_{pmax}}$ for Max Power:

Air Drag Force, Fa: $\mathrm{Fa}(v_v) := 0.5 \cdot \rho \cdot \mathrm{Ad} \cdot \left[(v_v + \mathrm{Vw})^2 \cdot \mathrm{Cd} + \mathrm{Cd_{cw}} \cdot (V_{cw})^2\right]$

Total Opposing Force, Fo: $\mathrm{Fo}(v_v) := \mathrm{Fa}(v_v) + \mathrm{Ft}(v_v)$ $\mathrm{Fo}(60 \cdot \mathrm{mph}) = 120.16 \cdot \mathrm{lbf}$

Note: For Drag and Road Resistance, approximate vehicle with v_{tire}. At < 60mph Compared to Ftire, Fo is small.

Torque/Force Falloff Curve: $T_{PLt}(\omega_k) := \mathrm{Power_{max}} \cdot \Omega(\omega_k)^{-1}$ $T_m(\omega_k) := \mathrm{if}(\omega_k \cdot \mathrm{RPM} \geq \omega_{Pfall}, T_{PLt}(\omega_k), T_{max})$

Tm is Torque of motor $T_{m1}(\omega_k) := \mathrm{Torque}(\omega_k \cdot 1000) \cdot \mathrm{lbf} \cdot \mathrm{ft}$ $P_m(\omega_k) := T_m(\omega_k) \cdot k \cdot 2 \cdot \pi \cdot \omega_k \cdot \mathrm{RPM}$ $P_m(8) = 8000 \, \mathrm{hp}$

Fmot, Tractive Force from motor, not from slipping tires: $T_{mv}(v_t) := T_m(\mathrm{VtokR}(v_t) \cdot \mathrm{GR})$ $F_{mot}(v_t) := \frac{\mathrm{GR}}{r_{tire}} \cdot T_{mv}(v_t)$ $F_{mot}(1 \mathrm{mph}) = 1.11 \times 10^4 \cdot \mathrm{lb}$

Solve for Velocity, Acceleration, and Distance versus Time

We are using Mathcad 14, a Computer Math Program, to do the Calculations: http://www.ptc.com/product/mathcad/free-trial

Case 1: Perfect Grip Tires at Maximum Motor Power, No limit on Coefficient of Tire Friction

Newton's Third Law of Motion:

$$a_1(v) := \frac{F_{mot}(v) - Fo(v)}{k_m \cdot M_{gross}} \qquad a_{1Tmax} := \frac{T_{max} \cdot GR}{M_{gross} \cdot k_m \cdot r_{tire}} = 4.33 \cdot g$$

$$V := 0 \cdot mph \qquad vel_1(t) := root\left(t \cdot sec - \int_0^V \frac{mph}{a_1(V \cdot mph)} \, dV, V \right) \cdot mph \qquad time_{a1}(v) := \int_0^v \frac{1}{a_1(v)} \, dv$$

$$time_{a1}(74mph) = 0.78\,s$$

$$vel_1(3.83) = 327.44 \cdot mpl$$

$$v_{gfall} := root\left(a_1(V \cdot mph) - car_{max\_g}, V\right) = 196.06 \qquad time_{a1}(60mph) = 0.64\,s \qquad a_{1t}(t) := a_1\left(vel_1(t)\right)$$

Velocity g fall, $a \le 1.4g$ $\quad a_1\left(v_{gfall}\,mph\right) = 4 \cdot g$ $\quad a_{1t}(0) = 4.32 \cdot g$

Case 2: High Performance 4 g Tires & Motor Drive Limited Accel < 4 g/ No Spin, but Max Power

a_2 acceleration is allowed by high performance tires on dry road.

$$a_2(v) := if\left(a_1(v) \ge car_{max\_g}, car_{max\_g}, a_1(v)\right) \qquad car_{max\_g} = 4 \cdot g$$

$$vel_2(t) := root\left(t \cdot sec - \int_0^V \frac{mph}{a_2(V \cdot mph)} \, dV, V \right) \cdot mph \qquad time_{a2}(v) := \int_0^v \frac{1}{a_2(v)} \, dv \qquad vel_2(1.8) = 157.95 \cdot mph$$

$$a_2\left(v_{gfall}\,mph\right) = 4 \cdot g$$

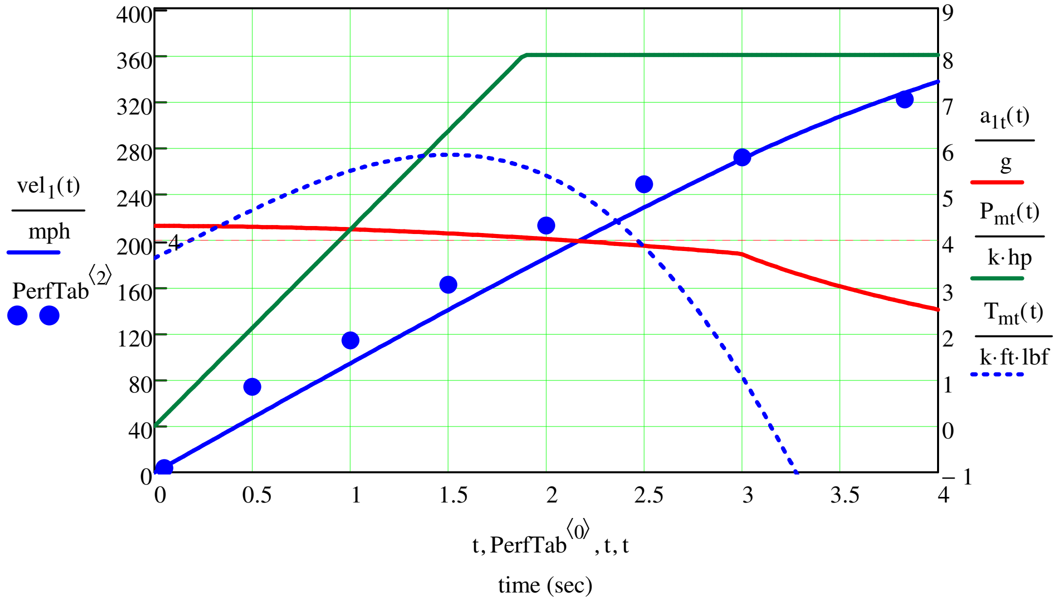
$$distance_2(t) := \int_0^t vel_2(t)\,\tau\,dt \qquad a_{2t}(t) := a_2\left(vel_2(t)\right) \qquad a_{2t}(0mph) = 4 \cdot g \qquad time_{a2}(60mph) = 0.68\,s$$

$$distance_2(7) = ■ \cdot mile \qquad t_{gfall} := time_{a2}\left(v_{gfall} \cdot mph\right) = 2.23\,s$$

RPM at g fall: $R_{gfall} := VtokR\left(v_{gfall} \cdot mph\right) \cdot GR$

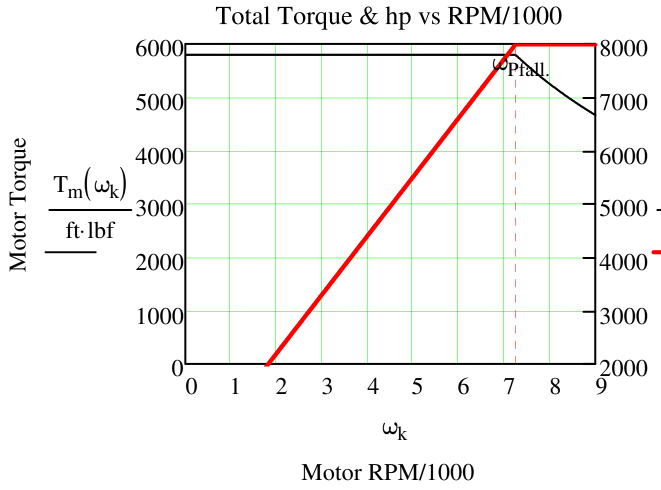
V. Model Results and Validation: Matches Data

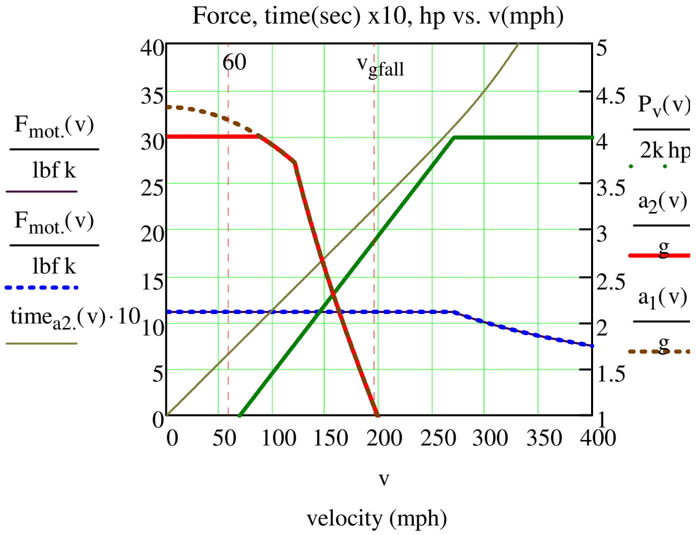
The difference between the data (dots) and Simulation Model is that the Model does not account for the aerodynamic down forces which give a boost in acceleration above 60 mph.

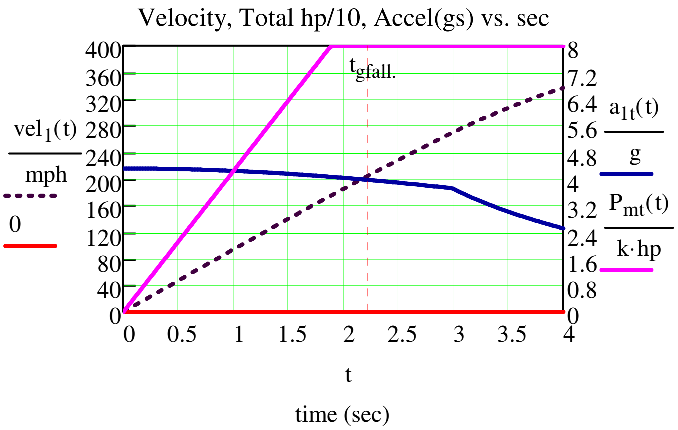

Velocity (Data (Dots), Model-Blue), Total hp/10, Accel(gs-Red) vs. sec

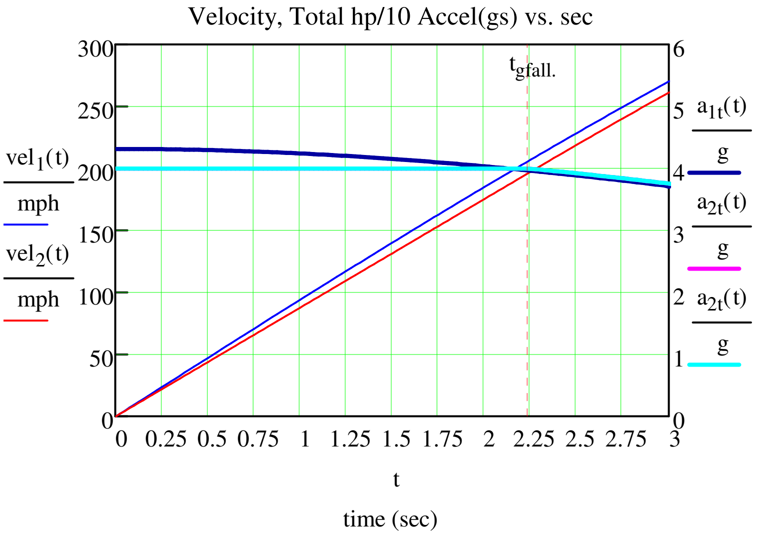
$$PerfTab = \begin{pmatrix} 0.05 & 0 & 4 \\ 0.5 & 10.27 & 73.89 \\ 1 & 52 & 113.82 \\ 1.5 & 125 & 162 \\ 2 & 232 & 213 \\ 2.5 & 379.9 & 248.5 \\ 3 & 566 & 271.6 \\ 3.83 & 1000 & 321.6 \end{pmatrix}$$

VI. Performance Curves









IX. Tire Friction (Composition and Width)

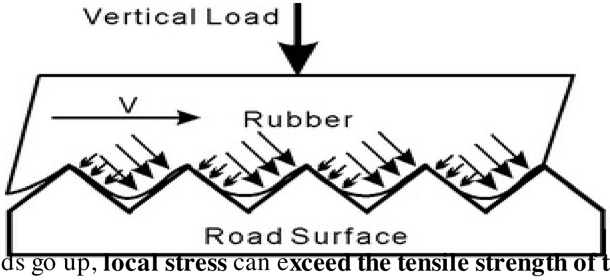
Coefficient of Static Friction (µ) is the ratio of Tire Road Force to Vehicle Weight. Values of µ for Conventional Car tire On: Asphalt 0.72, Car tire Grass 0.35.

Top Fuel drag car tires are getting a coefficient of friction well **over 4.5. How is this possible?**

This material came from: http://insideracingtechnology.com/tirebkexerpt1.htm **See Mathcad/EVs/Tire Friction.doc**

Rubber generates friction in three major ways: **adhesion, deformation, and wear.**

Rubber in contact with a **smooth surface** (glass is often used in testing) generates friction forces mainly by **adhesion**. **When rubber is in contact with a rough surface, another mechanism, deformation, comes into play. Movement** of a rubber slider on a r**ough surface** results in the **deformation of the rubber by high points o**n the surface called irregularities or **asperities**. A load on the rubbe slider causes the asperities to **penetrate the rubber** and the **rubber drapes over** the asperities. The **energy needed** to move the asperities in the rubber comes from the **differential pressure** across the asperities as shown in Fig. 3.4, where a rubber slider moves on an irregular surface at speed V.



Tearing and Wear

As deformation forces and sliding speeds go up, **local stress** can **exceed the tensile strength of the rubber**, especially at an increase in local stress near the point of a **sharp irregularity**. High local stress can **deform the internal structure of the rubber past the point of elastic recovery.** When polymer bonds and crosslinks are **stressed to failure the material can't recover completely**, and this can cause **tearing. Tearing absorbs energy**, resulting in **additional friction forces** in the contact surface.

Wear is the ultimate result of tearing.

Ftotal = Fadhesive + Fdefformation + Fwear

Deformation Friction and Viscoelasticity

Rubber is elastic and conforms to surface irregularities. But rubber is **also viscoelastic**; it **doesn't rebound fully** after deformation.

Hysteresis

Hysteresis, or energy loss, in rubber.

where there is **some sliding** between the rubber and an irregular surface. If the **rubber recovers slowly** from the passing irregularity as in the high-hysteresis rubber, it **can't push o**n the downstream surfaces of the irregularities **as hard** as it pushes on the upstream surfaces. This **pressure difference** between the **upstream and downstream faces of the irregularity** results in **friction forces** even when the surfaces are lubricated.

Wide Tires: It is true that wider tires commonly have better traction. The main reason why this is so does not relate to contact patch, however, but to **composition. Soft compound tires** are required to be **wider in order for the side-wall to support the weight** of the car softer tires have a larger coefficient of friction, therefore better traction. A narrow, soft tire would not be strong enough, nor would it last very long. **Wear in a tire is related to contact patch. Harder compound tires wear much longer**, and can be narrower. They do, however have a lower coefficient of friction, therefore less traction. Among tires of the same type and composition, here is no appreciable difference in 'traction' with different widths. **Wider tires**, assuming all other factors are equal, commonly have **stiffer side-walls and experience less roll. This gives better cornering performance.**

Friction is proportional to the normal force of the asphalt acting upon the car tires. This force is simply equal to the weight which is distributed to each tire when the car is on level ground. Force can be stated as Pressure X Area. **For a wide tire, the area is large but the force per unit area is small** and vice versa. The force of friction is therefore the same whether the tire is wide or not. However, **asphalt is not a uniform surface**. Even with steamrollers to flatten the asphalt, the surface is still somewhat irregular, especially over the with of a tire. Drag racers can therefore **increase the probability or likelihood of making contact** with the road by using a wider tir In addition a secondary benefit is that the wider tire increased the support base a

Friction force is independent of the apparent area of contact. **For hard materials**, this is nearly correct. The true area of contact varies with the applied load. The apparent area does not. If you can imagine the contact zone from a **microscopic viewpoint, only a tiny portion of the apparent area actually touches.** This tiny area is the true area of contact. But this applies to hard materials. It does **not apply to elastomers, such as rubber.** Tire tread rubber compounds vary greatly from one application to another. **Race car tire** tread compounds can be **very soft, viscoelastic materials**, while heavy truck tread rubber can be quite hard. In general, **soft rubber materials have greater friction**. With drag racing 'slicks,' the tire tread material **literally sticks** to the pavement--and the **rubber is sheared from the tire.** Clearly, the greater the apparent contact area, the greater this shear force. **Cleanliness is important** to getting the surfaces to '**stick.**' This is one reason why drag racers have a '**burn-out**' before each race (another is to raise the tire tread surface temperature). However, there is another reason for wide tire treads on some road and track racing cars. They need **tread volume to provide enough wear life**. Tires wear rapidly under racing conditions. **Some long races wear out several sets of tires.** There are **trade-offs with traction and tread life.** That is why heavy truck tire tread compounds do not have as much friction as those used on passenger cars. However, truck tire tread compounds provide longer wear life and less heat build-up. Like many things in this world, **tire tread choices involve compromises.**