

Extract Koren Parameters for KT150

Koren Phenomenological Vac. Tube Spice Models

http://www.normarkoren.com/Audio/Tubemodspice_article.html

Use Mathcad 14: Vacuum Tube SPICE Models.xmcd from VXPhysics.com/Integrated%20Circuits/

Koren SpiceTube Parameters

Read File Values into "Data":

Data :=

Koren-Tube Data.xls

Koren's Data does not include KT Spice Model

Data for **KT150**

TUBE	MU	EX	KG1	KG2	KP	KVB	CCG	CPG	CCP
6DJ8	28	1.3	330	0	320	300	2.3	2.1	0.7
6L6CG	8.7	1.35	1460	4500	48	12	14	0.85	12
6550	7.9	1.35	890	4800	60	24	14	0.85	12
KT88	8.8	1.35	730	4200	32	16	14	0.85	12

Definitions: MU is the Vp vs Vg Transconductance, KG1 is the Ip scale factor, 1/KG, KG2 is I_{G2} scale factor, 1/KG2, KP is a large factor in the Low IP-High Vp Region, KVB is knee voltage for low voltage Ip vs Vp region.

Koren Triode Model

Row for 12AX7: n := 3

Koren Params: $(\mu, Ex, k_{g1}, k_p, k_{vb}) := (Data_{n,1}, Data_{n,2}, Data_{n,3}, Data_{n,5}, Data_{n,6})$

Koren Model:

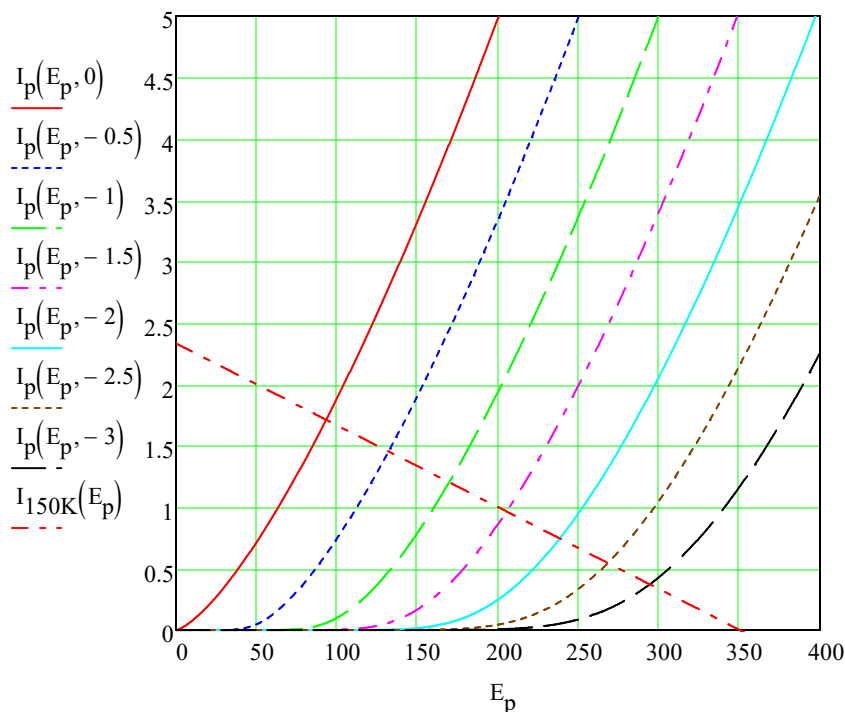
$$E_1(E_p, E_g) := \frac{E_p}{k_p} \ln \left[1 + \exp \left[k_p \cdot \left(\frac{1}{\mu} + \frac{E_g}{\sqrt{k_{vb} + E_p^2}} \right) \right] \right]$$

$$I_p(E_p, E_g) := 1000 \frac{E_1(E_p, E_g)^{Ex}}{k_{g1}} (1 + \text{sign}(E_1(E_p, E_g)))$$

150K Load Line: $I_{150K}(E_p) := (350 - E_p) \cdot \frac{1000}{150000}$

Plot Koren Triode Model - Fig. 1

Triode 12AX7: Avg. Plate Characteristics



Koren Pentode Model Ipp vs. Ep/Eg

Row for 6550 in Koren's Spice Data:

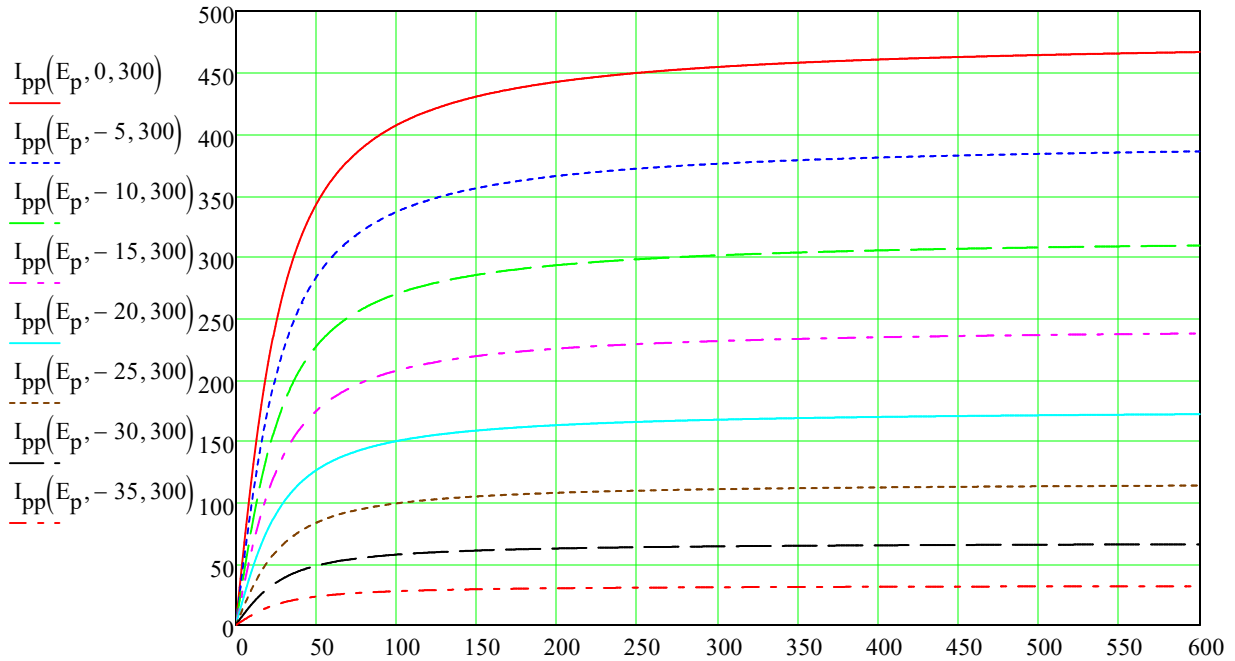
$n := 5$

$$(\mu, E_x, k_{g1}, k_p, k_{vb}) := (\text{Data}_{n,1}, \text{Data}_{n,2}, \text{Data}_{n,3}, \text{Data}_{n,5}, \text{Data}_{n,6})$$

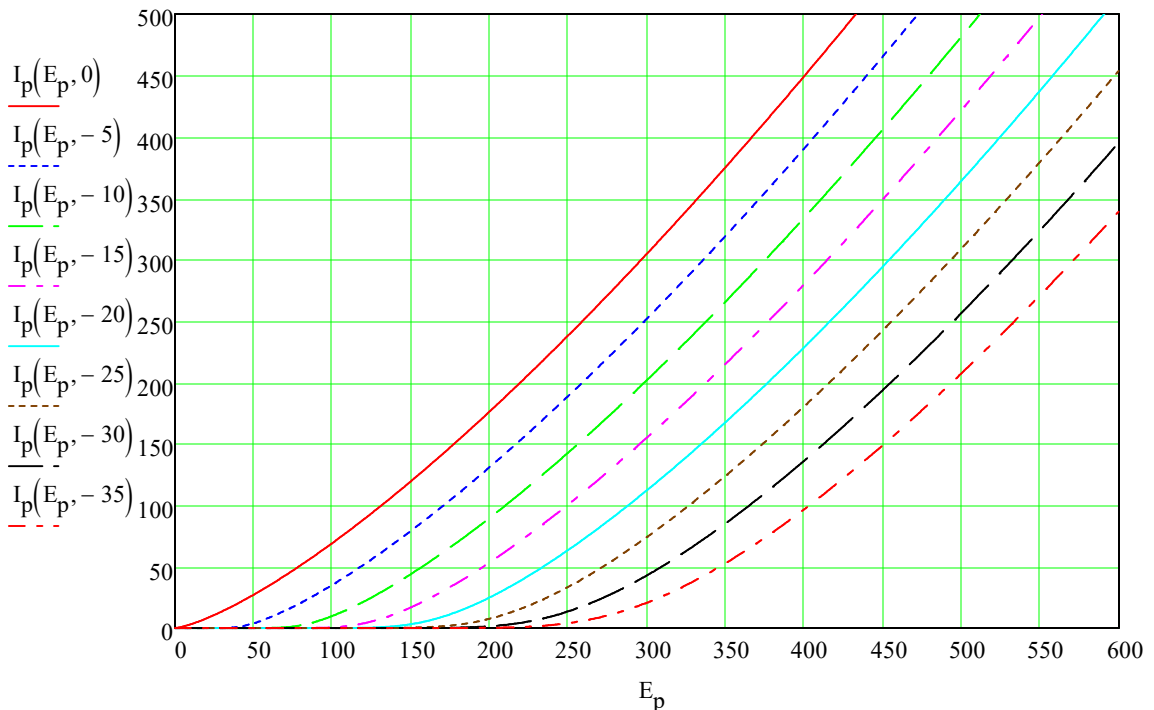
$$E_{1p}(E_g, E_{g2}) := \frac{E_{g2}}{k_p} \ln \left[1 + \exp \left[k_p \cdot \left(\frac{1}{\mu} + \frac{E_g}{E_{g2}} \right) \right] \right]$$

$$I_{pp}(E_p, E_g, E_{g2}) := 1000 \frac{E_{1p}(E_g, E_{g2})^{E_x}}{k_{g1}} (1 + \text{sign}(E_{1p}(E_g, E_{g2}))) \cdot \text{atan} \left(\frac{E_p}{k_{vb}} \right)$$

Pentode Mode 6550-A: Avg. Plate Characteristics (mA) - Fig. 2



Triode Mode 6550-A: Avg. Plate Characteristics (mA) - Fig. 3



Extract Koren SPICE Pentode Parameters for KT150

KT150 Tung-Sol Published Data

From: Frank's Electron Tube Pages

<https://frank.pocnet.net/sheets/084/k/KT150.pdf>

μ , Voltage Multiplier
 Plate Resistance, r_p , Approximately 3000 Ohms
 Mutual Transconductance, g_m 12.6 ma/V

$$\mu = \frac{\Delta E_p}{\Delta E_g}$$

$$r_p = \frac{\Delta E_p}{\Delta I_p}$$

$$g_m = \frac{\Delta I_p}{\Delta E_g}$$

KT150 Interelectrode Capacitance:

C_{gp} 1.75 pf
 C_{input} 20.5 pf
 C_{output} 10 pf

Therefore: $r_p = \frac{\mu}{g_m}$

We have a very limited data sample for Ip-Vp Curves

Note: Published Curve Data is given only at $V_{g2} = 225V$ and for just two tubes.

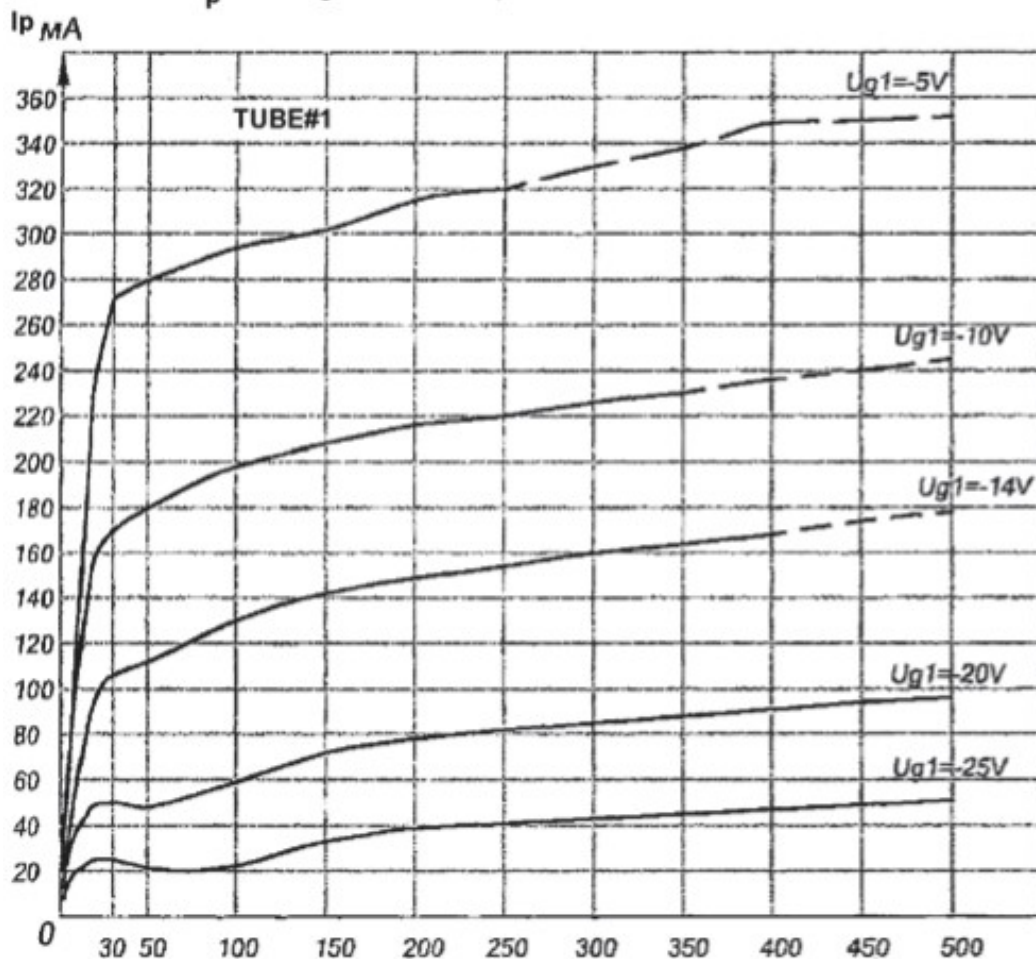
Source: www.newsensor.com www.ehx.com

Our Data is based on Tube#1, which has the Larger "Premium" Plate Current.

Note: The **LARGE SLOPE**, $\Delta I_p / \Delta V_p \sim 70mA/300V$ (Plate Conductance) @ $V_{g1} = -5V$.

KT150TS, $I_p = f(U_p)$ - Fig. 4

$U_p = 400V, U_{g2} = 225V, U_H = 6,3V$



Digitize Above Tung-Sol KT150 Tube#1 Curves and get Data Points

Used "PlotDigitizer_2.6.8_Windows" Program for Digitizing of Above KT150 Curves

Read Digitized KT150 Data from TWK Data File from Above 5Vg C Curves into Matrix KT

The only data we have is the pentode curves for a fixed screen voltage 225V and five Vgk, grid to cathode values. Points on the curve were digitized, with more points taken on those regions with the greatest bending to capture "kinks" and fewer on the linear regions. A total of 158 Ip - Vpk data point pairs for all the 5 Vgk KT150 curves were recorded and read into the variable KT. Approximately, 32 points per curve, which seemed to best represent the curve, were digitized. The error in the Ip ~300mA/-5V Vgk curve is estimated at ±1.5% and increases to about ± 5% for the smaller Ip ~ 40mA/-25V Vgk curve.

Using Paint.net graphics software to show the DIFFERENCE between layers of the Tung-Sol and the above digitized Images layer shows a very close image match.

```
KT := READPRN("KT150PlotData.txt")    rows(KT) = 47    cols(KT) = 10
```

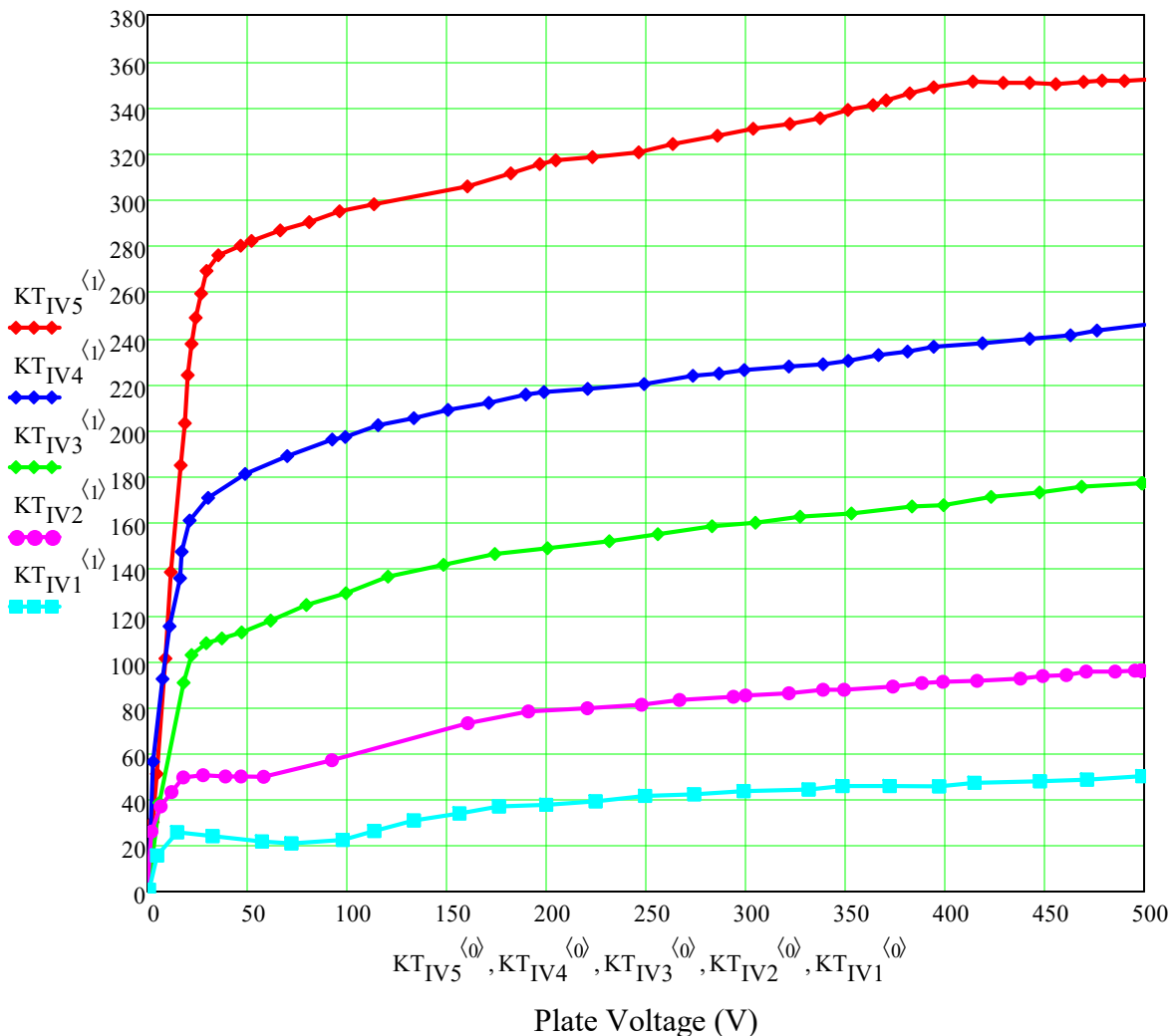
Define 5 Plate Current and Voltage Arrays for each of the 5 Grid Voltages: -5 -10, -14, -20, -25 Volts

```
KTIV1 := submatrix(KT,0,46,0,1)  KTIV2 := submatrix(KT,0,46,2,3)  KTIV3 := submatrix(KT,0,46,4,5)
```

```
KTIV4 := submatrix(KT,0,46,6,7)  KTIV5 := submatrix(KT,0,46,8,9)
```

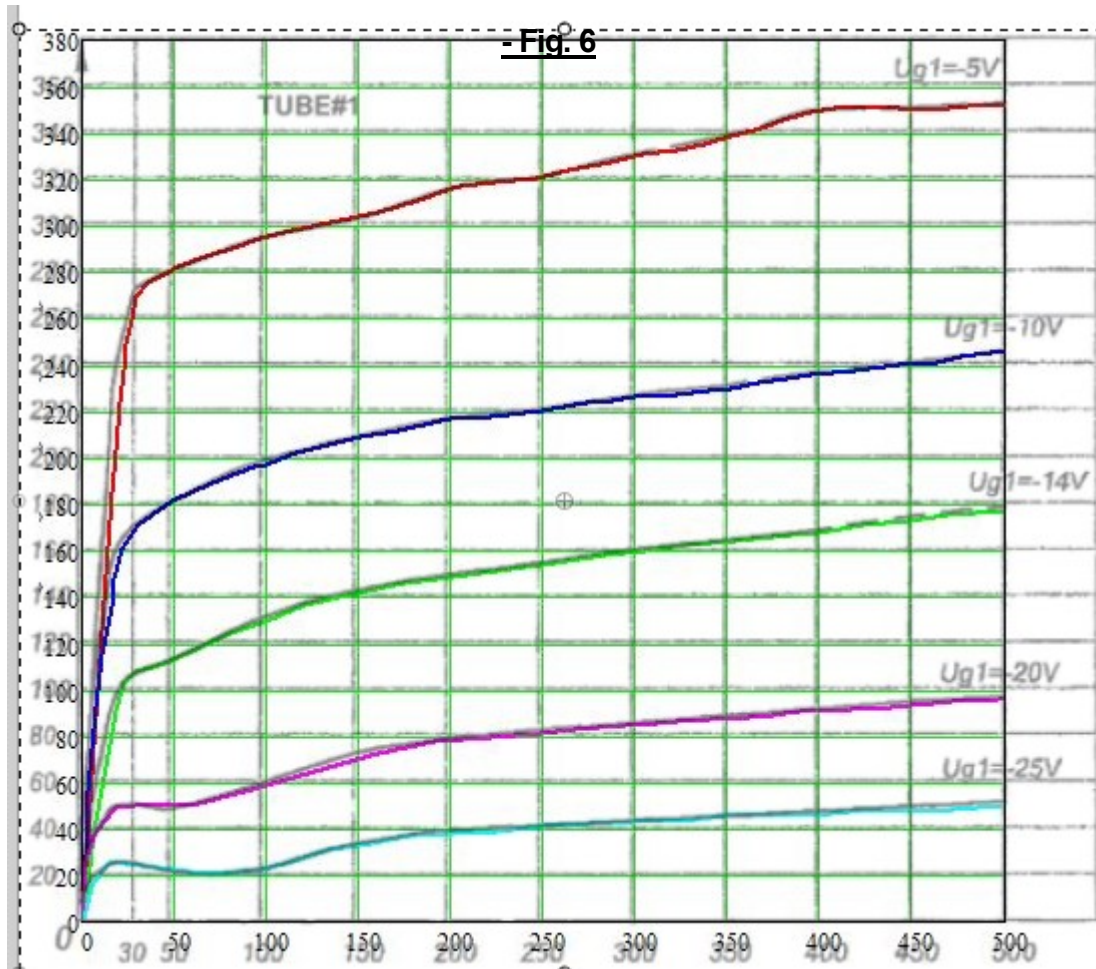
Below are Plots of the Digitized Data Vgk = -5, -10, -14, -20, -25V

Pentode Mode KT150TS: Avg. Plate Characteristics (mA) - Fig. 5



Original Tung-Sol Operating Curve Data Superimposed onto Digitized Curves

This image shows the Tung-Sol superimposed onto of the Digitized Curves.
Note very close match of digitized curves except at initial linear portion near origin



We note some small nonlinearities in the horizontal scale between images.
This may be caused by rotational difference between images.

Methodology to Extract KT150 Koren Parameters

Our methodology is similar to Konar's (reference 2), the major difference being we use Mathcad's error minimization function Minerr, and not MATLAB's fmins() error minimization tool.

Read All Data Points into a 3 Column Array of Vp, Ip, Vg.
Used Excel to remove any duplicate data points by Ip. Store
Columns into the Data File: "KT150DigPlotVIG.txt"

Read into the 3 Column Vp, Ip, Vg Array, KT_{VIG} :

$$KT_{VIG} := \text{READPRN}(\text{"KT150DigPlotVIG.txt"}) \quad \text{rows}(KT_{VIG}) = 158$$

Define Koren Plate Current Fitting Function I_{pf} for Above 5 Grid Voltages Curves

To reflect compatibility with Child-Langmuir V_{pk}^{3/2} Law for Thermionic Emission

Set Ex in the Model I_{pf}(E_x, μ, k_{xx}) to the Same Value as for a KT88, that is, Ex = 1.35

$$Vg2 = 225V \quad E_{1pf}(E_g, \mu, k_p) := \frac{225}{k_p} \ln \left[1 + \exp \left[k_p \cdot \left(\frac{1}{\mu} + \frac{E_g}{225} \right) \right] \right]$$

$$I_{pf}(E_p, E_g, \mu, k_{g1}, k_p, k_{vb}) := 1000 \frac{E_{1pf}(E_g, \mu, k_p)^{1.35}}{k_{g1}} \left(1 + \text{sign}(E_{1pf}(E_g, \mu, k_p)) \right) \cdot \text{atan} \left(\frac{E_p}{k_{vb}} \right)$$

Initial Guess (6550 Params): $(\mu, k_{g1}, k_p, k_{vb}) := (\text{Data}_{n,1}, \text{Data}_{n,3}, \text{Data}_{n,5}, \text{Data}_{n,6})$
 $n = 5$
 $(\mu, k_{g1}, k_p, k_{vb}) = (7.9, 890, 60, 24)$

Check Param Guess for Reasonableness: $I_{pf}(500, -5, \mu, k_{g1}, k_p, k_{vb}) = 242.627$

Use Levenberg-Marquardt Method: Minimize Least Squares Error to Plate Current

Define a Residual to be the difference between the current data points KT_{VIG} and Model I_{pf}(E_x, μ, k_{xx})

Given the Plate Voltage - Current - Grid Voltage in Matrix KT_{VIG}

$$\text{Residual}(\mu, k_{g1}, k_p, k_{vb}) := \text{KT}_{VIG}^{\langle 1 \rangle} - \text{I}_{pf}(\text{KT}_{VIG}^{\langle 0 \rangle}, \text{KT}_{VIG}^{\langle 2 \rangle}, \mu, k_{g1}, k_p, k_{vb})$$

Condition to Minimize the Residual Least Squares Fit Error Using L-V Minerr Method

Given

$$0 = \text{Residual}(\mu, k_{g1}, k_p, k_{vb})$$

Use Minerr to Extract Optimal Parameters: $(u, k_{g1}, k_p, k_{vb}) := \text{Minerr}(\mu, k_{g1}, k_p, k_{vb})^T$

Extracted Koren Params: $(u, k_{g1}, k_p, k_{vb}) = (10.774, 412.457, 28.64, 14.226)$

Put Koren SPICE Params for 5 V_g Curves into Array Sp: $Sp := (u, k_{g1}, k_p, k_{vb})^T$

SSE Error Estimate: $\text{ERR}_{\text{sse}} := \sqrt{\sum \text{Residual}(u, k_{g1}, k_p, k_{vb})^2} \cdot \frac{1}{158}$ **ERR_{sse} = 0.905 Average Error ~ 1 mA**

ERR is an Internal Variable that gives L-M SSE Error: $\frac{\text{ERR}}{158} = 0.905$ **ERR Same as SSE**

Define Plate Current vs Voltage Function: I_{pk}(E_p, V_g), using KT150 Koren Spice Params, Sp

$I_{pk}(E_p, V_g) := I_{pf}(E_p, V_g, Sp_0, Sp_1, Sp_2, Sp_3)$ **Check at 500V:** $I_{pk}(500, -5) = 338.95$

$I_{pk}(500, -10) = 229.687$ $I_{pk}(500, -14) = 158.873$ $I_{pk}(500, -20) = 82.13$ $I_{pk}(500, -25) = 43.014$

Compare Extracted KT150 Koren Model to Digitized Ip-Vp Curve Data Digitized Tube Data Shown as Points. Koren Model are Lines

Note Again, that these Parameters are based on a Very Limited Data Sample

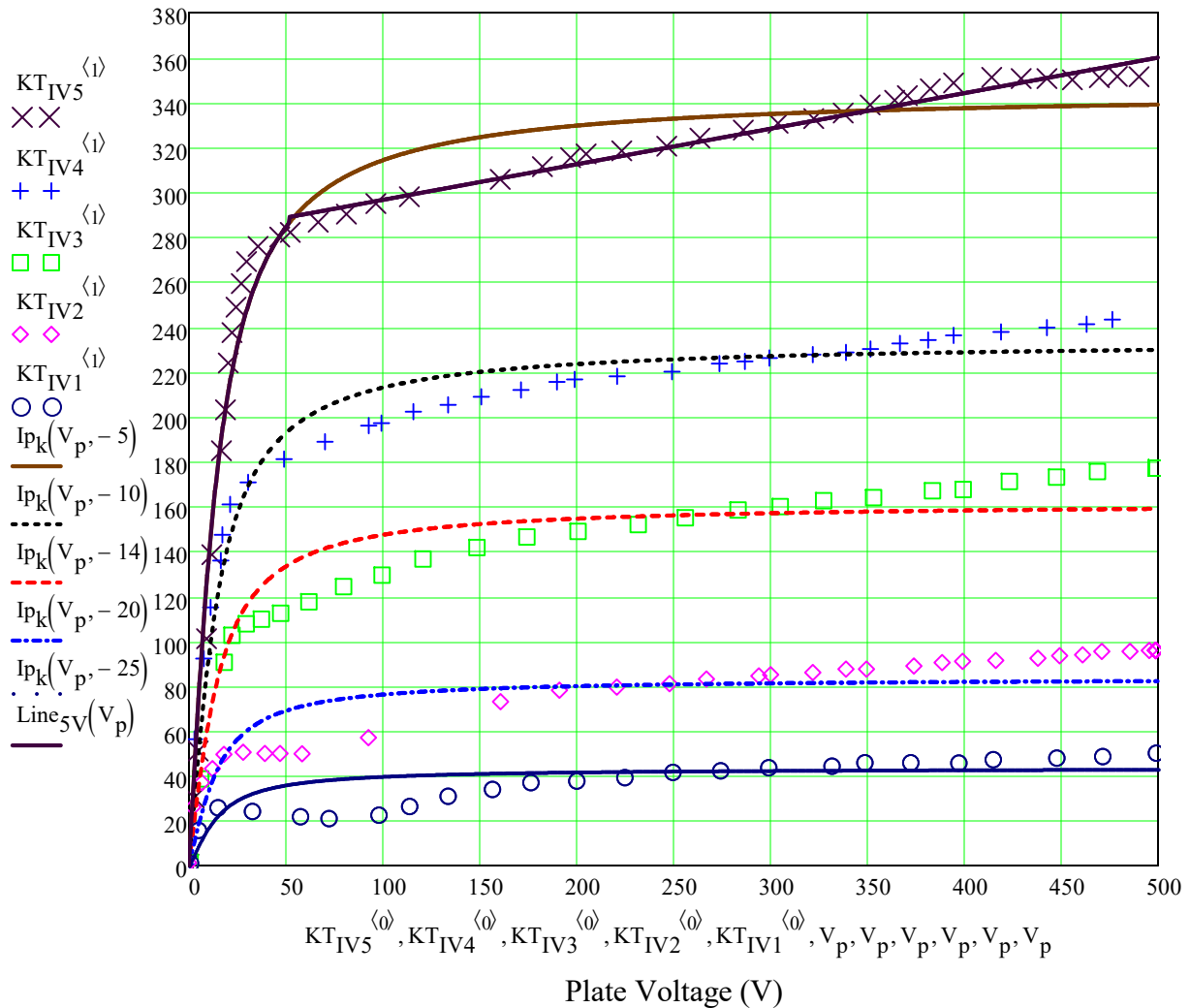
Extracted KT150 Koren Parameters: $(u \text{ kg1 } kp \text{ kvb}) = (10.774 \text{ } 412.457 \text{ } 28.64 \text{ } 14.226)$

Find the line that best matches the Slope of the -5V Digitized Data curve between Vp 50 to 400V. Create an Array VI5A for the digitized VI values of -5V data from 100 to 400V.

$$ab := \text{line}(VI5A^{(0)}, VI5A^{(1)}) \quad \text{Line}_{5V}(Vp) := \text{if}(Vp > 51.6, ab_0 + ab_1 \cdot Vp, Ip_k(Vp, -5))$$

$V_p := 0, 1..500$

Pentode Mode KT150TS: Avg. Plate Characteristics (mA) - Fig. 7



Inspection of the above KT150 plots show that except for matching the Ip-Vp Plate Conductance line, Line5V(Vp),

there is a reasonably good match between Koren's SPICE model and the digitized data.

Lets do an Improved Model of the ΔIp/ΔVp Slope

Characterize Slope of $\Delta I_p/\Delta V_p$ or Plate Conductance, G_p

We will use the above digitized KT curves to measure the slope ($\Delta I/\Delta V$) of the curves. Select data in the KT curves between **100 and 400V** Vpk and put them into **Arrays** named **Vlxx**, where xx is the -gate voltage. Then the plate resistance, $\Delta V/\Delta I$, is 1/slope of I_p - V_p curve. Thus for Vlxx, the function $r_p(VI)$ below gives rp.

Plate conductance, G_p , is defined as ratio of the change of plate current to change in plate voltage

$$G_p = \frac{\Delta I_p}{\Delta V_p}$$

The Mathcad slope() function gives the best fit value to the VI values

$$G_p(VI) := \text{slope}(VI^{(0)}, VI^{(1)})$$

Plate resistance, r_p , is defined as ratio of the change of plate voltage to change in plate current

$$r_p = \frac{\Delta E_p}{\Delta I_p}$$

for constant grid to plate voltage.

Since transconductance, μ , is defined as

$$\mu = \frac{\Delta E_p}{\Delta E_g}$$

for constant I_p .

and mutual conductance, g_m , is defined as

$$g_m = \frac{\Delta I_p}{\Delta E_g}$$

for constant E_p .

then r_p is equal to

$$r_p = \frac{\mu}{g_m}$$

We want to Characterize the $\Delta I/\Delta V$ slope (Plate Conductance, G_p) of the lines in the digitized data

The Gate voltages (V_{gA}) for Vlxx Arrays of KT150 Digitized Data are: $V_{gA} := (5 \ 10 \ 14 \ 20 \ 25)^T$

Plate Conductance, G_p , ($\Delta I/\Delta V$) slope **between 50 and 500V** for the Vlxx arrays is given by the "slope" function" $G_p(VI)$, The above will give units of mA/volt. This is very small.

To **make the units more manageable**, give this in **units of mA/100V**;

$$G_p(VI) := \text{slope}(VI^{(0)}, VI^{(1)}) \cdot 100$$

$$G_p(VI5A) = 15.844 \quad G_p(VI10A) = 12.482 \quad G_p(VI14A) = 13.088 \quad G_p(VI20A) = 8.655 \quad G_p(VI25A) = 6.436$$

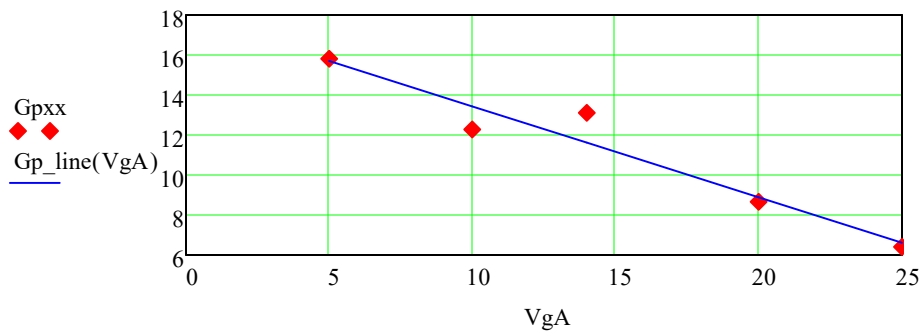
Put the above values into the Array G_{pxx} $G_{pxx} := (15.8 \ 12.28 \ 13.1 \ 8.65 \ 6.4)^T$

Fit these to the best fit line and plot. $GM := \text{line}(V_{gA}, G_{pxx})$

$$G_{p_line}(V_g) := GM_0 + GM_1 \cdot V_g$$

We can Approximate Plate Conductance, G_p , vs. Grid Voltage with a Linear Model!

Plate Conductance vs Grid Voltage (ma/100V)



$$GM^T = (17.974 \ -0.455)$$

Similarly, we can get the plate resistance, $r_p = \Delta V/\Delta I$, which is the reciprocal of the slope (1/slope) of the I_p - V_p curve.

$$r_p(VI) := \frac{1}{\text{slope}(VI^{(0)}, VI^{(1)})}$$

Use the above $r_p(VI)$ slope function to calculate the Array Plate Resistance in k ohms for best fit line. From below, we see that r_p increases with V_g and thus decreases with Plate Current, I_p :

$$r_p(VI5A) = 6.311 \quad r_p(VI10A) = 8.012 \quad r_p(VI14A) = 7.641 \quad r_p(VI20A) = 11.554 \quad r_p(VI25A) = 15.537$$

Put the above r_p values into the Array R_p

$$R_p := (6.3 \ 8 \ 7.6 \ 11.5 \ 15.5)^T$$

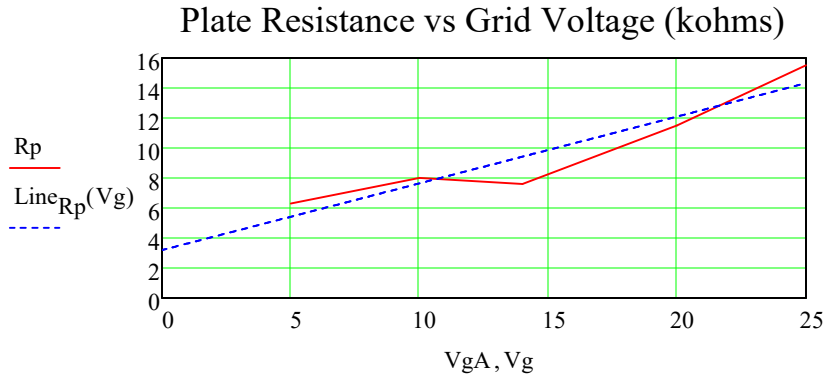
Consistency Check

Check to see if the Rp values calculated above are consistent with the earlier calculation of Gp

$$G_{p_g} := \left(\frac{1}{R_p} \cdot 100 \right)^T = (15.873 \quad 12.5 \quad 13.158 \quad 8.696 \quad 6.452)$$

Get the best line fit, $Line_{Rp}$: $AB := \text{line}(VgA, Rp)$ $AB^T = (3.19 \quad 0.445)$ $Line_{Rp}(Vg) := AB_0 + AB_1 \cdot Vg$

Fit the best fit line and plot $Line_{Rp2}(Vg) := AB_1 \cdot Vg$

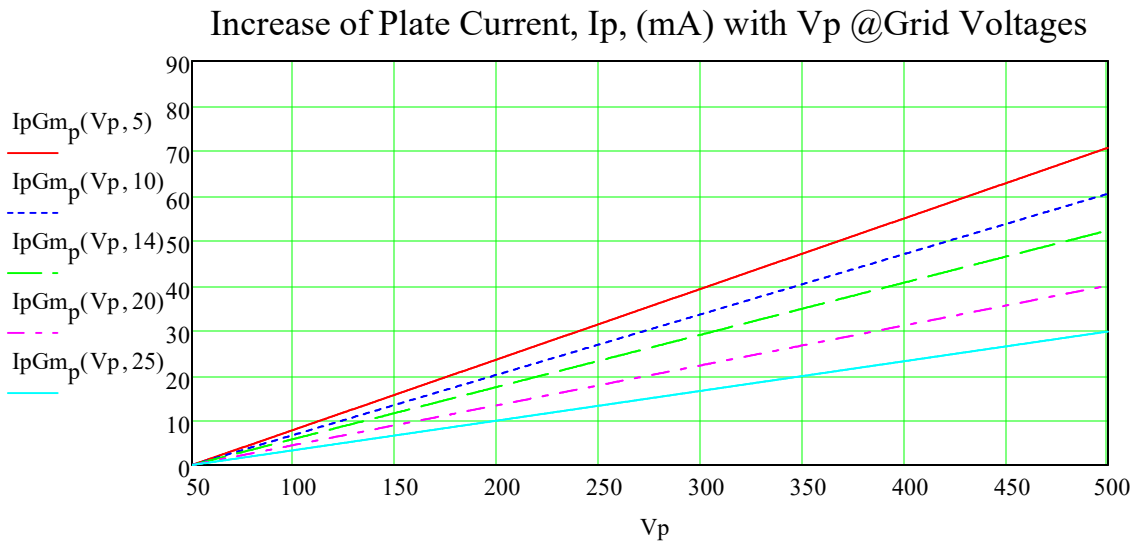


We can reasonably represent the Large Signal Conductance ($\Delta I/\Delta V$) slope of the KT100 curves for the different Gate Voltages with a Linear Model. Using SSE to find best fit parameters should improve the model. We can do either the above Linear Gate Voltage Models for the Plate Conductance, Gp, or for the Plate Resistance, rp. Since a Vacuum Tube is basically a Voltage Controlled Current source, we will use the Plate Conductance, Gp, Model.

We Model the Tube with Gp for the ΔI_p versus V_p as function of V_g beyond $V_p=50V$.

GM is (Intercept, slope) for Gp: $GM^T = (17.974 \quad -0.455)$ $Gp_line(Vg) := GM_0 + GM_1 \cdot Vg$

$$IpGm_p(V_p, V_g) := \frac{(V_p - 50) \cdot Gp_line(V_g)}{100}$$



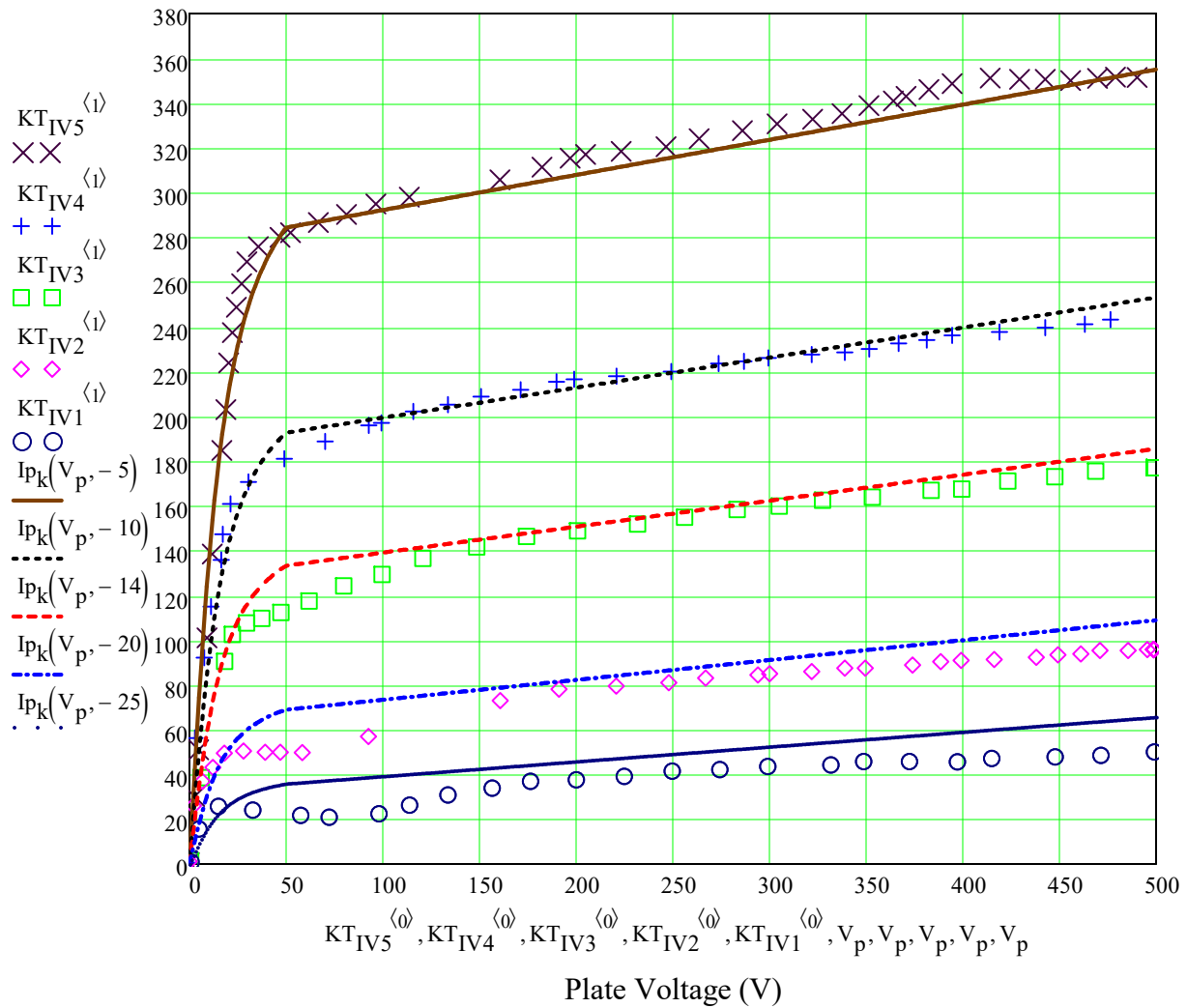
Linear Model for Plate Conductance. G_p , from Above

$$G_{p_line}(V_g) := GM_0 + GM_1 \cdot V_g$$

$$I_{pGm_p}(V_p, V_g) := \frac{(V_p - 50) \cdot G_{p_line}(V_g)}{100}$$

$$I_{p_k}(V_p, V_g) := \text{if}(V_p < 50, I_{p_k}(V_p, V_g), I_{p_k}(50, V_g) + I_{pGm_p}(V_p, -V_g))$$

Pentode Mode KT150TS: Avg. Plate Characteristics (mA) - Fig. 7



$$\text{ResidNew} := KT_{VIG}^{(1)} - \overline{I_{p_k}(KT_{VIG}^{(0)}, KT_{VIG}^{(2)})}$$

$$\text{ERR3} := \sqrt{\sum \text{ResidNew}^2} \cdot \frac{1}{158} = 0.773$$

Percent Error Improvement Over Koren:

$$\frac{0.773 - 0.905}{0.905} = -14.586\%$$

The Koren Linear Plate Conductance Model Representation

Define Koren Plate Current Fitting Function Ipf for Above 5 Grid Voltages Curves

To reflect compatibility with Child-Langmuir $Vpk^{3/2}$ Law for Thermionic Emission

Set Ex to the Same Value as for a KT88, that is, Ex = 1.35

$$Vg2 = 225V \quad E_{1pf}(E_g, \mu, k_p) := \frac{225}{k_p} \ln \left[1 + \exp \left[k_p \cdot \left(\frac{1}{\mu} + \frac{E_g}{225} \right) \right] \right]$$

$$I_{p0}(E_p, E_g, \mu, k_{g1}, k_p, k_{vb}) := 1000 \frac{E_{1pf}(E_g, \mu, k_p)^{1.35}}{k_{g1}} \left(1 + \text{sign}(E_{1pf}(E_g, \mu, k_p)) \right) \cdot \text{atan} \left(\frac{E_p}{k_{vb}} \right)$$

$$I_{pGmp}(V_p, V_g, G_{p0}, G_{p1}) := (V_p - 50) \cdot (G_{p0} + G_{p1} \cdot V_g) \cdot 0.001$$

To avoid small numbers, multiply Gp by 0.001, then Gp is in units of milli Siemens, mS

$$I_{lin}(E_p, E_g, \mu, k_{g1}, k_p, k_{vb}, G_{p0}, G_{p1}) := I_{p0}(E_p, E_g, \mu, k_{g1}, k_p, k_{vb}) + I_{pGmp}(E_p, -E_g, G_{p0}, G_{p1})$$

$$I_{pl}(E_p, E_g, \mu, k_{g1}, k_p, k_{vb}, G_{p0}, G_{p1}) := \text{if}(E_p < 50, I_{p0}(E_p, E_g, \mu, k_{g1}, k_p, k_{vb}), I_{lin}(E_p, E_g, \mu, k_{g1}, k_p, k_{vb}, G_{p0}, G_{p1}))$$

Initial Guess (6550 Params):

$$\begin{aligned} \mu &:= 5 \\ (\mu, k_{g1}, k_p, k_{vb}, G_{p0}, G_{p1}) &:= \left(\text{Data}_{n,1}, \text{Data}_{n,3}, \text{Data}_{n,5}, \text{Data}_{n,6}, 95, \frac{-1}{1} \right) \\ (\mu, k_{g1}, k_p, k_{vb}, G_{p0}, G_{p1}) &= (7.9, 890, 60, 24, 95, -1) \end{aligned}$$

Check Param Guess for Reasonableness: $I_{pl}(500, -5, \mu, k_{g1}, k_p, k_{vb}, G_{p0}, G_{p1}) = 283.127$

Use Levenberg-Marquardt Method: Minimize Least Squares Error to Plate Current

$$\text{Resid}(\mu, k_{g1}, k_p, k_{vb}, G_{p0}, G_{p1}) := \overrightarrow{KT_{VIG}^{(1)}} - I_{pl} \left(\overrightarrow{KT_{VIG}^{(0)}}, \overrightarrow{KT_{VIG}^{(2)}}, \mu, k_{g1}, k_p, k_{vb}, G_{p0}, G_{p1} \right)$$

Condition for Minimum Residual Least Squares Fit Error Using L-V Minerr Method

Given

$$0 = \text{Resid}(\mu, k_{g1}, k_p, k_{vb}, G_{p0}, G_{p1})$$

Use Minerr to Extract

Optimal Parameters:

$$(\mu, k_{g1}, k_p, k_{vb}, G_{p0}, G_{p1}) := \text{Minerr}(\mu, k_{g1}, k_p, k_{vb}, G_{p0}, G_{p1})^T$$

Extracted Koren Params:

$$(\mu, k_{g1}, k_p, k_{vb}, G_{p0}, G_{p1}) = (9.953, 484.887, 37.964, 11.302, 94.978, -1.339)$$

ERR is an Internal Variable that gives L-M SSE Error:

Percent Error Improvement Over Koren:
This model has 61% less SSE error than Koren's

$$\frac{\text{ERR}}{\text{rows}(KT_{VIG})} = 61.24\%$$

Define Plate Current vs Voltage Function: Ipk(Ep, Vg), using KT150 Koren Spice Params, Sp

Put Koren SPICE Params for 5 V_g Curves into Array Sp:

$$Sp := (\mu, k_{g1}, k_p, k_{vb}, G_{p0}, G_{p1})^T$$

$$Sp^T = (9.953, 484.887, 37.964, 11.302, 94.978, -1.339)$$

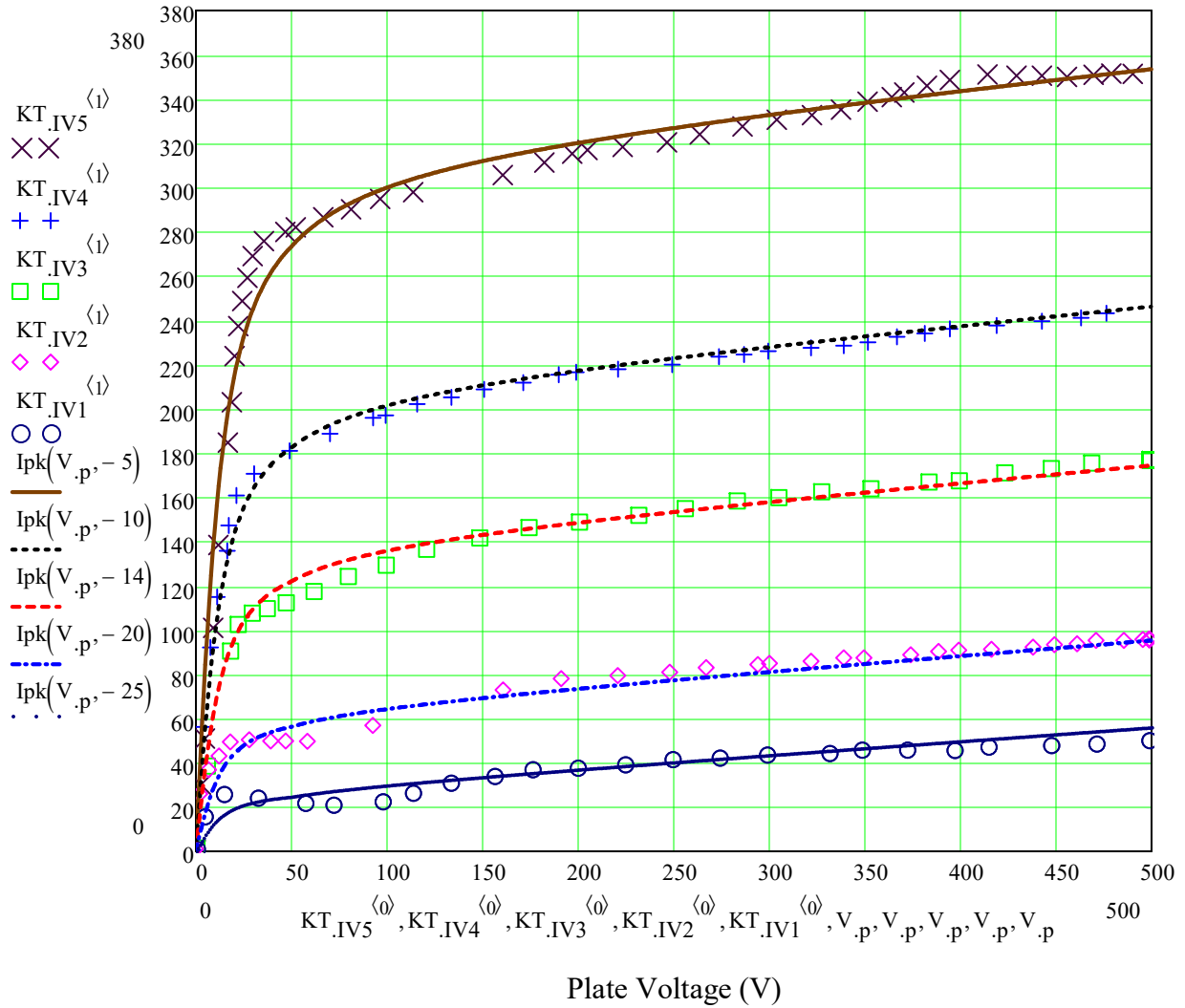
$$I_{pk}(E_p, V_g) := I_{pl}(E_p, V_g, \mu, k_{g1}, k_p, k_{vb}, G_{p0}, G_{p1})$$

Check at 500V: $I_{pk}(500, -5) = 353.51$

$$I_{pk}(500, -10) = 246.233 \quad I_{pk}(500, -14) = 174.418 \quad I_{pk}(500, -20) = 95.299 \quad I_{pk}(500, -25) = 56.206$$

Koren Linear Plate Conductance Model, KT150 Gp

Pentode Mode KT150TS: Avg. Plate Characteristics (mA) - Fig. 7



Model Error Analysis

The greatest Residual error between the digitized curve and model is @Vg-5V between Vp 20 to 80V. This is also greatest error source for my digitization of the Initial Tung-Sol Curves Source Data.

Summary

Koren Type Phenomological Model Parameters: KT150 and KT150-Gp

TUBE	MU	EX	KG1	KG2	KP	KVB	Gpo	Gp1	CCG	CPG	CCP	RGI
Units						V	mS	mS	pF	pF	pF	Ohm
6550	7.90	1.35	890	4800	60.0	24.00			14	0.85	12	1000
KT88	8.80	1.35	730	4200	32.0	16.00			14	0.85	12	1000
KT150	10.77	1.35	412.46	4200	28.6	14.23			14	0.85	12	1000
KT150-Gp	9.95	1.35	485	4200	38.0	11.30	95	-1.34	14	0.85	12	1000